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RESEARCH ANNOUNCEMENTS

The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.

PRODUCTS OF SYMMETRIES

BY PAUL R. HALMOS AND SHIZUO KAKUTANI

Communicated January 25, 1958

A (bounded) operator Q on a (complex) Hilbert space H is a *symmetry* if it is a unitary involution, i.e., if $Q^*Q=QQ^*=1$ (=the identity operator on H) and $Q^2=1$. In connection with his studies of the infinite-dimensional analogues of the classical groups, R. V. Kadison has asked us which operators can be represented as (finite) products of symmetries. The purpose of this note is to give a precise answer to Kadison's question.

THEOREM 1. *If H is infinite-dimensional, then every unitary operator on H is the product of four symmetries.*

PROOF. We need the auxiliary result that if U is a unitary operator on an infinite-dimensional Hilbert space H, then there exists a (closed) subspace H_0 of H such that H_0 reduces U and such that $\dim H_0 = \dim H_0^\perp$. This result holds, in fact, for an arbitrary normal operator on H. Since the proof is a straightforward application of the spectral theorem, and since the proof for a typical special case (namely, for Hermitian operators) has already appeared in the literature,[1] we do not present it here.

We apply the auxiliary result to the (unitary) operator on $H_0^\perp$ obtained by restricting U to $H_0^\perp$ and obtain thus a subspace H_1 (of $H_0^\perp$) such that H_1 reduces U and such that $\dim H_1 = \dim (H_0^\perp \cap H_1^\perp)$. Proceeding inductively, we obtain an infinite sequence $\{H_n\}$ of orthogonal subspaces (of H) such that each H_n reduces U and such that every H_n has the same dimension. If the intersection of the orthogonal complements of all the H_n is not trivial, it can be amalgamated to H_0; it follows that H is the direct sum of countably many equidimensional subspaces each of which reduces H. By suitably renumbering the terms of this sequence, we may assume that the index n runs through all (not necessarily non-negative) integers.

Relative to the fixed direct sum decomposition $H=\sum_n H_n$, we define a *right shift* as a unitary operator S such that $SH_n = H_{n+1}$ for all n, and we define a *left shift* as a unitary operator T such that $TH_n = H_{n-1}$ for all n. The equi-dimensionality of all the H_n guarantees the existence of shifts. If S is an arbitrary right shift, we write $T = S^*U$. Since $TH_n = S^*UH_n = S^*H_n = H_{n-1}$ for all n, it follows that T is a left shift. Since $U = ST$, we have proved that every unitary operator on H is a product of two shifts; we shall complete the proof of the theorem by showing that every shift is the product of two symmetries.

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[1] Paul R. Halmos, *Commutators of operators*, Amer. J. Math. vol. 74 (1952) pp. 237–240; see Lemma 3 on p. 239.

Since the inverse (equivalently, the adjoint) of a left shift is a right shift, it is sufficient to consider right shifts. Suppose then that S is a right shift; let P be the operator that is equal to S^{1-2n} on H_n and let Q be the operator that is equal to S^{-2n} on H_n for all n. If $x \in H_n$, then $Qx = S^{-2n}x \in S^{-2n}H_n = H_{-n}$, so that $PQx = PS^{-2n}x = S^{1-2(-n)}S^{-2n}x = Sx$. The proof of Theorem 1 is complete.

To what extent is Theorem 1 the best possible result along these lines? The hypothesis of infinite-dimensionality clearly cannot be omitted. Indeed, if H is finite-dimensional, then the concept of determinant makes sense. Since the determinant of a symmetry is ± 1, it follows that no (unitary) operator with a nonreal determinant can be the product of symmetries. Equally clearly, the conclusion of the theorem cannot be strengthened so as to apply to nonunitary operators, because a product of unitary operators (and, in particular, of symmetries) must be unitary. The only conceivable improvement, therefore, is quantitative: possibly every unitary operator is a product of three symmetries. We conclude by showing that this is not so.

THEOREM 2. *On every Hilbert space H there exists a unitary operator U that is not the product of three symmetries.*

PROOF. Let c be a complex cube root of unity and let U be $c \cdot 1$. The operator U belongs to the center of the group of all unitary operators on H; the order of U in that group is exactly three. The remainder of our proof has nothing to do with operator theory; we shall show that, in every group G, a central element of order three is not the product of three elements of order two. More precisely, we show that if G is a group, if u is a central element in G, and if $u = xyz$ with $x^2 = y^2 = z^2 = 1$, then $u^4 = 1$. The proof consists of a simple computation:

$$\begin{aligned} u^4 &= uxuyuz = u(xu) \cdot y(uz) = u(yz) \cdot y(xy) \\ &= y(uz) \cdot y(xy) = yxy \cdot yxy = 1. \end{aligned}$$

UNIVERSITY OF CHICAGO AND
YALE UNIVERSITY

ON THE WHITEHEAD HOMOMORPHISM J

BY JOHN MILNOR[1]

Communicated by R. H. Fox, February 3, 1958

Consider the homomorphism $J\colon \pi_{r-1}(SO_n) \rightarrow \pi_{n+r-1}(S^n)$ of G. W. Whitehead[2] in the stable range $n>r$. The object of this note is to prove:

THEOREM 2. *Let q be an odd prime and let r be any multiple of $2(q-1)q^i$, $i \geqq 0$. Then for $n>r$ the image $J\pi_{r-1}(SO_n) \subset \pi_{n+r-1}(S^n)$ contains a cyclic subgroup of order q^{i+1}.*

According to recent work of Adams (as yet unpublished) the stable group $\pi_{n+r-1}(S^n)$ has the following q-primary components:

Z_q for $r=2i(q-1)$, $i<q$ (this result is due to Cartan);
Z_q for $r=2q(q-1)-1$;
Z_{q^2} for $r=2q(q-1)$; and zero for other values of r less than $2q(q-1)$.

Comparing this with Theorem 2 we have:

COROLLARY. *For $r<2q(q-1)-1$, and for $r=2q(q-1)$, the image $J\pi_{r-1}(SO_n)$ contains the q-primary component of the stable group $\pi_{n+r-1}(S^n)$.*

The corresponding assertion for $r=2q(q-1)-1$ is false, since the group $\pi_{r-1}(SO_n)$ is zero[3] in this case.

The proof will be based on work of Thom, Hirzebruch, Borel and von Staudt.

THEOREM 1. *Let ξ be the SO_n-bundle over S^r corresponding[4] to an element λ of $\pi_{r-1}(SO_n)$. If $J\lambda=0$ then there exists an oriented manifold M^r, differentiably imbedded in the sphere S^{n+r}, and having the following property: Some map $g\colon M^r \rightarrow S^r$ of degree $+1$ is covered by a bundle map of the normal bundle of M^r into the given bundle ξ.*

(It is not asserted that M^r is connected.) The manifolds M^r constructed in this way will be further studied in a later paper.[5]

PROOF OF THEOREM 1. Let E be the total space of the n-cell bundle over S^r associated with ξ; so that the boundary $\dot{E}$ is the total space of the associated $(n-1)$-sphere bundle. Consider the identification space $E/\dot{E}$ obtained by collapsing $\dot{E}$ to a point e^0. This space has a cell subdivision with cells e^0, e^n, and e^{n+r}; where e^n corresponds to the inverse image of a base point in S^r. The Thom isomorphism[6] $\Phi: H^i(S^r) \to H^{i+n}(E, \dot{E})$ can be used to give specific orientations to the cells e^n, e^{n+r}. In any such cell complex the cell e^{n+r} is attached to the sphere $e^0 \cup e^n$ by means of an attaching map $S^{n+r-1} \to e^0 \cup e^n$ which is well defined up to homotopy.

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[1] The author holds a Sloan fellowship.

[2] G. W. Whitehead, Ann. of Math. vol. 43 (1942), pp. 634–640.

[3] See R. Bott, Proc. Nat. Acad. Sci. U.S.A. vol. 43 (1957) pp. 933–935.

[4] See Steenrod, *The topology of fibre bundles*, 1951, p. 99.

[5] J. Milnor, *A generalization of a theorem of Rohlin*, to appear.

LEMMA 1. *The homotopy class of the attaching map, considered as an element of* $\pi_{n+r-1}(S^n)$, *is equal to* $J\lambda$.

The proof, which is not difficult, will be given in a subsequent paper.[7]

Now suppose that $J\lambda = 0$. Then the complex $E/\dot{E}$ has the homotopy type of the union $S^n \vee S^{n+r}$ with a single point in common; hence there exists a map $f: S^{n+r} \to E/\dot{E}$ of degree $+1$.

The complement $E/\dot{E} - e^0$ can be considered as a differentiable manifold, with submanifold S^r corresponding to the trivial cross-section of the cell bundle. Following Thom[8] the map f can be approximated by a map f_1 which is t-regular on S^r. The inverse image $M^r = f_1^{-1}(S^r)$ is then a differentiable manifold with a canonical orientation. Furthermore, if $g: M^r \to S^r$ denotes the restriction of f_1, then g is covered by a bundle map of the normal bundle of M^r into the given bundle ξ.

Let T_1 and T_2 be suitably chosen open tubular neighborhoods of M^r and S^r. From the commutativity of the diagram

$$\begin{tikzcd}
H^r(S^r) \arrow[r, "\Phi"] \arrow[d, "g^*"] & H^{n+r}(E/\dot{E}, E/\dot{E} - T_2) \arrow[r] \arrow[d] & H^{n+r}(E/\dot{E}) \arrow[d, "f_1^*"] \\
H^r(M^r) \arrow[r, "\Phi"] & H^{n+r}(S^{n+r}, S^{n+r} - T_1) \arrow[r] & H^{n+r}(S^{n+r})
\end{tikzcd}$$

it follows that g has degree $+1$. This completes the proof of Theorem 1.

Now suppose that r is equal to $4k$. The Pontrjagin classes p_i of the manifold M^r are clearly zero for $i < k$. Furthermore the Pontrjagin number $p_k[M^r]$ is equal to $-\langle p_k(\xi), \mu\rangle$, where μ denotes the standard generator of $H_r(S^r; Z)$. For such a manifold the Hirzebruch index formula[9] reduces to $\tau(M^r)=s_k p_k[M^r]$; where $s_1=1/3$, $s_2=7/45, \cdots$, and in general s_k equals $2^{2k}(2^{2k-1}-1)/(2k)!$ times the Bernoulli number B_k. Since $\tau(M^r)$ is an integer by definition this implies:

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[6] R. Thom, Ann. Sci. Ecole Norm. Sup. vol. 69 (1952) pp. 109–181.

[7] J. Milnor, *On spaces with a gap in cohomology*, Theorem 3, Corollary 1, to appear.

[8] R. Thom, Comment. Math. Helv. vol. 28 (1954) pp. 17–86.

COROLLARY 1. *Let* ξ *and* λ *be as above, with* $r=4k$. *The condition* $J\lambda=0$ *implies that* $s_k\langle p_k(\xi), \mu\rangle$ *is an integer.*

Borel and Hirzebruch[10] have constructed an example of an SO_n-bundle ξ_0 over S^{4k}, $n>4k$, such that the number $\langle p_k(\xi_0), \mu\rangle$ is equal to $2(2k-1)!$. Let λ_0 be the corresponding element of $\pi_{4k-1}(SO_n)$ and let h be the order of its image $J\lambda_0$. (h is a positive integer since the stable homotopy groups of spheres are known to be finite groups.) Then for the bundle ξ_1 corresponding to $h\lambda_0$ the number

$$s_k\langle p_k(\xi_1), \mu\rangle = s_k h 2(2k-1)!$$

must be an integer. An immediate consequence is the following.

COROLLARY 2. *The order* h *of the element* $J\lambda_0$ *of* $\pi_{n+4k-1}(S^n)$ *is a multiple of the denominator of the rational number* $2(2k-1)!\, s_k$, *expressed as a fraction in lowest terms.*

The author is indebted to Hirzebruch for calling his attention to the following two theorems, which will be used to compute the above denominator. An odd prime q is said to be "of rank k" if $2k\equiv 0$ (modulo $q-1$). Let b_k denote the product of all odd primes of rank k.

First theorem of von Staudt.[11] *The denominator of the Bernoulli number* B_k, *expressed as a fraction in lowest terms, is equal to* $2b_k$.

Thus, setting $B_k=a_k/2b_k$, the integer a_k is odd and has no prime factor of rank k. Any positive integer k can be expressed as a product $k=2^t k_1 k_2$ with k_1k_2 odd, where all of the prime factors of k_1 are of rank k, while none of the prime factors of k_2 is of rank k.

Second theorem of von Staudt.[11] *The numerator* a_k *of* B_k *is congruent to zero modulo* k_2.

Now the number $2(2k-1)!\, s_k=2^{2k}(2^{2k}-2)B_k/2k$ can be written as

$$2^{2k-t-2}(2^{2k}-2)(a_k/k_2)/b_k k_1.$$

The numerator and denominator of this expression are clearly integers. Furthermore the prime factors of the denominator are all odd primes of rank k. But no such prime divides the numerator. (The condition $2k \equiv 0 \pmod{q-1}$ implies that $2^{2k}-2 \equiv -1 \pmod{q}$.) Thus we have proved:

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[9] F. Hirzebruch, *Neue topologische Methoden in der algebraischen Geometrie*, 1956, p. 85.

[10] A. Borel and F. Hirzebruch, *Characteristic classes and homogeneous spaces*, to appear.

[11] See for example N. Nielsen, *Traité élémentaire des nombres de Bernoulli*, 1923, pp. 240–250.

Lemma 2. *The number* $2(2k-1)!\, s_k$, *when expressed as a fraction in lowest terms, has denominator* $b_k k_1$.

Proof of Theorem 2. If $r=4k$ is a multiple of $2(q-1)q^i$, it follows that q is of rank k. Hence q divides b_k and q^i divides k_1; so that q^{i+1} divides the denominator of $b_k k_1$. Together with Theorem 1, Corollary 2 this completes the proof.

THE CARTESIAN PRODUCT OF A CERTAIN NONMANIFOLD AND A LINE IS E^4

BY R. H. BING[1]

Communicated February 25, 1958

An upper semicontinuous decomposition G of E^3 into points and tame arcs is defined in [1] such that the decomposition space B is topologically different from E^3. Interesting properties of this space have also been given by Fort [4], Curtis [2; 3], and Wilder [3]. We show that the cartesian product of the space B and a line E^1 is topologically E^4. Perhaps the argument used is related to that employed by Arnold Shapiro to show that the cartesian product of a manifold described by Whitehead in [5] and a line is topologically E^4.

The arcs of the decomposition G are intersections of double tori as shown in the figure. The solid double torus contains four double tori T_1, T_2, T_3, T_4 as shown; each T_i in turn contains four double tori T_{i1}, T_{i2}, T_{i3}, T_{i4} (not shown) imbedded in T_i as T_1, T_2, T_3, T_4 were imbedded in T; more double tori are imbedded in the T_{ij}'s; etc. The tame arcs of the decomposition G are the components of

$$T \cdot \Sigma T_i \cdot \Sigma T_{ij} \cdot \Sigma T_{ijk} \cdot \cdots .$$

Although these tame arcs are mutually exclusive, it is not possible to get a 2-sphere in E^3 that misses their sum and separates two of them. No topological cube in T contains $T_1+T_2+T_3+T_4$.

When the cartesian product is taken, the extra dimension enables one to unravel certain linking handles in the sense that if $[a, b]$ is

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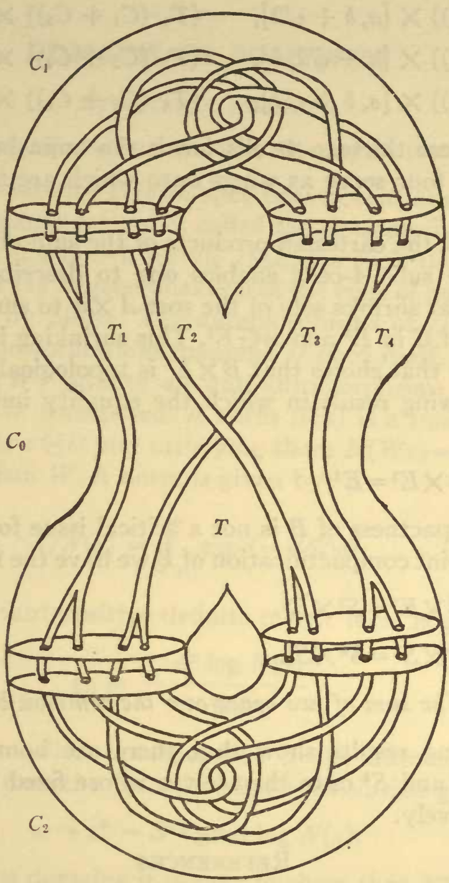
[1] Work supported by the National Science Foundation under NSF Grant G3248 at the Institute for Advanced Study, Princeton, New Jersey.



an interval and $\epsilon > 0$, then there is a topological 4-cell in

$$T \times [a-\epsilon, b+\epsilon]$$

that contains $(T_1+T_2+T_3+T_4) \times [a, b]$.

In order to describe this topological 4-cell, we use eight disks to chop T into seven topological cubes C_0, C_1, C_2, C_3, C_4, C_5, C_6 as shown in the figure. The topological 4-cell in $T \times [a-\epsilon, b+\epsilon]$ that contains $(T_1+T_2+T_3+T_4) \times [a, b]$ is the sum of the following thirteen 4-cells:

$C_0 \times [a-\epsilon, b+\epsilon]$,

$(C_1+C_3) \times [a-\epsilon, a-\epsilon/2]$, $\quad (C_2+C_4) \times [a-\epsilon, a-\epsilon/2]$,

$(C_1+C_5) \times [b+\epsilon/2, b+\epsilon]$, $\quad (C_2+C_6) \times [b+\epsilon/2, b+\epsilon]$,

$(T_1 \cdot (C_1+C_3)) \times [a-\epsilon/2, b]$, $\quad (T_2 \cdot (C_1+C_3)) \times [a-\epsilon/2, b]$,

$$(T_3\cdot(C_1+C_5))\times[a,b+\epsilon/2],\qquad (T_4\cdot(C_1+C_5))\times[a,b+\epsilon/2],$$
$$(T_1\cdot(C_2+C_4))\times[a-\epsilon/2,b],\qquad (T_3\cdot(C_2+C_4))\times[a-\epsilon/2,b],$$
$$(T_2\cdot(C_2+C_6))\times[a,b+\epsilon/2],\qquad (T_4\cdot(C_2+C_6))\times[a,b+\epsilon/2].$$

The first of these thirteen 4-cells forms the main body of the sum, while the next four serve as wings onto which are fastened the last eight.

The fact that the cartesian products of the sum of double tori and intervals lie in such 4-cells enables one to describe in isotopy on $E^3\times E^1=E^4$ that shrinks sets of the sort $A\times w$ to small size where A is a tame arc of G in E^3 and $w\in E^1$. This shrinking is continued to a pseudo isotopy that shows that $B\times E^1$ is topologically E^4. Hence we have the following result in which the equality implies topological equivalence.

THEOREM. $B\times E^1=E^4$.

The noncompactness of B is not a critical issue for if we let $\overline{B}$ denote the one point compactification of B we have the following results.

THEOREM. $\overline{B}\times E^1=S^3\times E^1$.

THEOREM. $\overline{B}\times S^1=S^3\times S^1$.

THEOREM. *The sum of two cones over the common base $\overline{B}$ is S^4.*

The preceding results show that there are homeomorphisms of period 2 of E^4 and S^4 onto themselves whose fixed point sets are B and $\overline{B}$ respectively.

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UNIVERSITY OF WISCONSIN AND
INSTITUTE FOR ADVANCED STUDY

DOMAINS OF POSITIVITY

BY OSCAR S. ROTHAUS

Communicated by S. Bochner, February 10, 1958

A Domain of Positivity D is an open convex cone associated with a nonsingular symmetric matrix S, called the characteristic, such that $x \in D$ if and only if $x'Sy > 0$ for all $y \in \bar{D}$. As such they were introduced by Koecher (1) in generalization of the cone of positive definite matrices studied by Siegel. The automorphisms of D are the nonsingular linear transformations mapping D onto itself. The group of automorphisms $\{W\}$ admits an anti-automorphism: $W \rightarrow S^{-1}W'S$, where W' means W transposed. A norm $N(x)$ is a function positive and continuous for $x \in D$ and satisfying there $N(Wx) = ||W|| N(x)$ for every automorphism W. A norm is given by:

$$1/N(x) = \int_D \exp(-x'St)dt$$

and a group invariant positive definite metric form is given by:

$$g_{ij} = -\frac{\partial^2 \log N(x)}{\partial x_i \partial x_j}.$$

The Domain is called homogeneous if the automorphisms are transitive. In this case the Domain has an involution given by:

$$x \rightarrow x^* = S^{-1} \text{ grad } \log N(x).$$

For homogeneous domains it is easy to show that $N^2(x)$ is always a rational function. If the characteristic is positive definite much more is true. In the first instance, the fixed points of the anti-automorphism of the group of automorphisms already act transitively on the domain D. It follows that the norm satisfies the important equality:

$$N(x^* + y^*) \cdot N(x) \cdot N(y) = N(x + y).$$

Moreover, for every point x in D there is an involution of the Domain keeping x fixed. Hence D is a symmetric (Cartan) space, and it is possible to make a detailed study of the Lie group of automorphisms. The following facts emerge:

(a) $N^2(x)$ is a polynomial,

(b) The geodesic connecting any two points (given by Cartan's construction of geodesics in a symmetric space) is unique,

(c) The Domain has everywhere zero or negative curvature.

(d) The involution on D extends to an analytic involution of the whole of the tube with D as base onto itself.

On the basis of (a) it follows from a result of Bochner (2) that the so-called "Gamma-Factor" of the Domain is indeed a product of Gamma functions. From (d) together with the equality described above satisfied by the norm, it follows from another result of Bochner (3) that there exist unitary transformations of $L_2(D)$ relative to the volume element $N^s(x)$ for suitable values of s, which generalize the Hankel Transform.

In quite another direction is the result, true for any homogeneous domain that, except for a multiplicative constant, $1/N^2(z+\bar{w})$ is the Bergmann reproducing kernel for analytic L_2 in the tube with D as base. This result shows quite clearly that the involution on D cannot extend to a holomorphic mapping of the tube with D as base unless the norm satisfies the equality stated earlier.

We understand Koecher has proved some of the same results, though they are not yet published at this writing.

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PRINCETON UNIVERSITY

ON THE PARALLELIZABILITY OF THE SPHERES

BY R. BOTT AND J. MILNOR

Communicated by H. Samelson, February 13, 1958

(The following note consists of excerpts from two letters.)

(*Milnor to Bott*; December 23, 1957.)

. . . Hirzebruch tells me that you have a proof of his conjecture that the Pontrjagin class p_k of a GL_m-bundle over the sphere S^{4k} is always divisible by $(2k-1)!$. I wonder if you have noted the connection of this result with classical problems, such as the existence of division algebras, and the parallelizability of spheres.

According to Wu the Pontrjagin classes of any GL_m-bundle, reduced modulo 4, are determined by the Stiefel-Whitney classes of the bundle. (See *On the Pontrjagin classes* III, Acta Math. Sinica vol. 4 (1954) in Chinese.) The proof makes use of the Pontrjagin squaring operation, together with the coefficient homomorphism $i: Z_2 \rightarrow Z_4$. Although I do not know the exact formula which Wu obtains, the following special case is not hard to prove:

LEMMA. *If the Stiefel-Whitney classes* $w_1, w_2, \cdots, w_{4k-1}$ *of a* GL_m-*bundle are zero then the Pontrjagin class* p_k, *reduced modulo* 4, *is equal to* $i_* w_{4k}$.

For a bundle over S^{4k} this means that w_{4k} is zero if and only if p_k is divisible by 4. Now if you can prove that p_k is divisible by $(2k-1)!$ it will follow that w_{4k} must be zero, whenever $k \geqq 3$.

THEOREM. *There exists a* GL_m-*bundle over* S^n *with* $w_n \neq 0$ *only if* n *equals* 1, 2, 4 *or* 8.

PROOF. Wu has shown that such a bundle can only exist if n is a power of 2. But the above remarks show that the cases $n = 16, 32, \cdots$ cannot occur.

COROLLARY 1. *The vector space* R^n *possesses a bilinear product operation without zero divisors only for* n *equal to* 1, 2, 4 *or* 8.

PROOF. Given such a product operation the map $S^{n-1} \rightarrow GL_n$ defined by $x \rightarrow$(left multiplication by x) gives rise to a GL_n-bundle over S^n for which it can be shown that $w_n \neq 0$.

COROLLARY 2. *The sphere* S^{n-1} *is parallelizable only for* $n-1$ *equal to* 1, 3 *or* 7.

PROOF. Given linearly independent vector fields $v_1(x), \cdots, v_{n-1}(x)$, on S^{n-1}, the correspondence

$$x \to (x, v_1(x), \cdots, v_{n-1}(x))$$

carries S^{n-1} into the Stiefel manifold of n-frames in R^n. Identifying this space with GL_n, we again obtain a GL_n-bundle over S^n with $w_n \neq 0 \cdots$

(*Bott to Milnor*, January 6, 1958.)

. . . Here is what I can show:

THEOREM. *Let $B = B_U$ be the universal base-space of the infinite unitary group. Then the image of $\pi_{2n}(B)$ in $H_{2n}(B)$ is divisible by precisely $(n-1)!$.*

This then refines the result of Borel-Hirzebruch that these classes are divisible by $(n-1)!$ except for the prime 2, [3], and confirms their conjecture. Because the Pontryagin classes are in the last analysis pre-images of classes in B_U, it follows that for any $GL_n(R)$ bundle over S^{4k}, p_k is divisible by $(2k-1)!$. This is all you needed.

The precise divisibility of p_k, for a real bundle over S^{4k}, is actually given by:

$$p_k = 0, \quad \begin{matrix} \text{mod } (2k-1)!, & k \text{ even}, \\ \text{mod } (2k-1)!2, & k \text{ odd}. \end{matrix}$$

This is seen by considering the fibering $U/O \to B_O \to B_U$.

The theorem follows from the fact, that if $\Omega = \Omega SU$ is the loopspace on SU, then there exists a homotopy equivalence $f: B \to \Omega$, as was announced in [1] and is proved in [2]. By standard theory the double suspension, S, from Ω into B, defines a homomorphism $\pi_k(\Omega) \to \pi_{k+2}(B)$ which is bijective for dimensions $\geqq 1$.

Let $\lambda = f_* \circ S$. It is then clear that:

$$(1) \qquad \pi_{2k}(\Omega) = \lambda^{k-1}(\pi_2(\Omega)).$$

Now in [2] the Hopf algebra $H_*(\Omega)$ is described. It turns out that $H_*(\Omega) = Z[\sigma_1, \sigma_2, \cdots]$, $\dim \sigma_i = 2i$, the diagonal map being: $\Delta_* \sigma_i = \sum \sigma_s \otimes \sigma_t$; $s + t = i$; $\sigma_0 = 1$. Hence the primitive subspace, P_*, is generated by elements $\{p_n\}$, $n = 1, 2, \cdots$, which are inductively determined by the relation:

$$(2) \qquad p_n - p_{n-1} \cdot \sigma_1 + p_{n-2} \cdot \sigma_2 - \cdots \pm n\sigma_n = 0, \qquad n = 1, 2, \cdots.$$

Let λ_* be the homomorphism corresponding to λ in homology. It will preserve spherical classes, and annihilate decomposable elements. It therefore follows from (2) that $\lambda_* p_i = \pm i \lambda_* \sigma_i$. As the spherical classes generate P_* (over the rationals, SU is a product of odd spheres!) this relation implies that $\lambda_*^{k-1}(H_2(\Omega))$ is divisible by at least $(k-1)!$. By (1) it follows that the spherical classes in dimension $2k$ are divisible by at least $(k-1)!$. This is the best bound because it is not hard to see that λ_* is *not* divisible on all of $H_*(\Omega)$.

An easy corollary of the theorem is that $\pi_{2n}(U_n)=Z/n!Z$. Kervaire also has decided the parallelizability question. He uses this formula as his starting point . . . .

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UNIVERSITY OF MICHIGAN AND
OXFORD UNIVERSITY

AN UNSHELLABLE TRIANGULATION OF A TETRAHEDRON

BY MARY ELLEN RUDIN

Communicated by R. H. Bing, February 14, 1958

A triangulation K of a tetrahedron T is shellable if the tetrahedra $K_1, \cdots, K_n$ of K can be so ordered that $K_i \cup K_{i+1} \cup \cdots \cup K_n$ is homeomorphic to T for $i=1, \cdots, n$. Sanderson [Proc. Amer. Math. Soc. vol. 8 (1957) p. 917] has shown that, if K is a Euclidean triangulation of a tetrahedron then there is a subdivision K' of K which is shellable; and he raises the question of the existence of a Euclidean triangulation of a tetrahedron which is unshellable. Such a triangulation will be described here.

Let T be a tetrahedron each of whose edges has length 1.

We will describe a nontrivial Euclidean triangulation K of T such that, if R is any tetrahedron of K, then the closure of $(T-R)$ is not homeomorphic to T.

I. *Construction of K*: Let X_1, X_2, X_3, and X_4 be the vertices of T.

The possible values for the letters i and j are 1, 2, 3, and 4 and addition involving i or j will be modulo 4.

For each i, let F_i denote the face of T opposite X_i, and let U_i be the midpoint of the interval X_iX_{i+2}. Observe that $U_1 = U_3$ and $U_2 = U_4$.

Let ϵ be the length of the shortest side of a triangle whose longest side is of length 1 and two of whose angles are 1° and 60°.

For each i, let Y_i denote the point of F_{i+1} at a distance $(3^{1/2}/2)\epsilon$ from X_i such that the angle $Y_iX_iX_{i+2}$ is 1°.

For each i, let Z_i denote the point of F_{i+2} such that the angle $Z_iX_iX_{i+3}$ is 1° and the angle $Z_iX_{i+1}X_i$ is 1°.

The fourteen vertices of our triangulation K are the points X_i, Y_i, Z_i, and U_i. It can be shown that no triangulation which has less than 14 vertices has the desired property.

The tetrahedra of our triangulation K are the tetrahedra of the forms:

(1) $X_iZ_iX_{i+1}Y_i$,
(2) $X_iZ_{i+1}X_{i+1}Y_i$,
(3) $Z_iZ_{i+1}X_{i+1}Y_i$,
(4) $Z_iZ_{i+1}X_{i+1}Y_{i+1}$,
(5) $Z_1Z_2Z_3Z_4$,
(6) $Z_iZ_{i+1}Y_iZ_{i+2}$,
(7) $X_iZ_{i+1}Y_iZ_{i+2}$,
(8) $X_iZ_{i+1}Y_{i+2}Z_{i+2}$,
(9) $X_iU_iY_{i+2}Z_{i+2}$,

(10) $X_iU_iY_iZ_{i+2}$,

(11) $Z_iU_iY_iZ_{i+2}$.

II. *Checking the construction*: The best method of doing this is to draw a big picture and label the vertices.

It is easy to check that for each tetrahedron R of K, the closure of $(T-R)$ is not homeomorphic to T.

In order to check that K is a triangulation, first observe that, for each i, the tetrahedra (1), (2), (3), and (4) fit together and form a thin rod having the triangles $X_iY_iZ_i$ and $X_{i+1}Y_{i+1}Z_{i+1}$ for its ends; the union of these rods forms a torus running along the edges X_iX_{i+1}. When (5) is added to this torus the remainder of T is divided into two congruent pieces each containing pieces of T along the faces F_i and F_{i+2}. After (6), (7), and (8) are added to the first five types there is only a small strip around X_iX_{i+2} remaining of T; (9) and (10) complete the faces of T and (11) fills in the final space.

To see that the tetrahedra all nest together properly in the order just described, the following facts will be useful. Fact A is needed for the "rods." Fact B is needed for (3). Facts C and D are needed as assurance that none of the tetrahedra of types (5) through (11) intersect the interior of the torus. Fact E is needed to show that (7) does not intersect either (2) or (6). And facts F, G, and H are needed to show that the tetrahedra of types (6) through (11) for $i=1$ do not intersect the tetrahedra of the same types for $i=3$. The facts can be easily proved using the definitions of ϵ, Y_i, and Z_i.

(A) The plane $X_iY_iZ_i$ separates X_{i+1}, Y_{i+1}, Z_{i+1} from X_{i-1}, Y_{i-1}, and Z_{i-1}.

(B) The points X_i and Y_i are on the same side of the plane $X_{i+1}Z_iZ_{i+1}$.

(C) The plane $Y_iZ_iZ_{i+3}$ separates X_i and X_{i+3} from U_i, X_{i+2}, Y_{i+2}, Z_{i+2}, X_{i+1}, Y_{i+1}, and Z_{i+1}.

(D) The plane $Y_iZ_iZ_{i+1}$ separates X_i and X_{i+1} from U_i, X_{i+2}, Y_{i+2}, Z_{i+2}, X_{i+3}, Y_{i+3}, and Z_{i+3}.

(E) The plane $X_iY_iZ_{i+1}$ separates Z_i from Z_{i+2}, X_{i+2}, Y_{i+2} and U_i.

(F) The plane $Z_iZ_{i+2}U_i$ separates X_i, Y_i, Z_{i+1}, Y_{i+1}, X_{i+1} from X_{i+2}, Y_{i+2}, Z_{i+3}, Y_{i+3}, X_{i+3}.

(G) The plane $Y_{i+2}Z_{i+2}U_i$ separates X_i and Z_{i+1} from X_{i+2}, X_{i+3}, Y_{i+3}, and Z_{i+3}.

(H) The plane $X_iZ_{i+2}U_i$ separates Y_{i+2} and Z_{i+1} from Y_i, Z_i, and Z_{i+3}.

UNIVERSITY OF ROCHESTER

TWO ELEMENT GENERATION OF THE PROJECTIVE UNIMODULAR GROUP[1]

BY A. A. ALBERT AND JOHN THOMPSON

Communicated February 21, 1958

In volume 31 (1930) of the Annals of Mathematics H. R. Brahana gave pairs of generators of the known simple groups whose orders are less than one million, and showed that one of the generators can be taken to have period two. In this note we shall outline our proof of the corresponding result for the general case of the projective unimodular group.

Let $\mathfrak{F} = \mathfrak{F}_q$ be the field of $q = p^n$ elements, $\mathfrak{M}(n, q)$ be the multiplicative group of all n-rowed square matrices with elements in $\mathfrak{F}$ and determinant unity, $\mathfrak{N} = \mathfrak{N}(n, q)$ be the center of $\mathfrak{M}(n, q)$, that is, the set of all scalar matrices of determinant unity. Then the projective unimodular group $\mathfrak{G}(n, q) = \mathfrak{M}(n, q)/\mathfrak{N}(n, q)$ is known to be simple. We have proved that it is generated by two cosets $A\mathfrak{N}$ and $B\mathfrak{N}$, where $A\mathfrak{N}$ has period two.

Let e_{ij} be the n-rowed square matrix with 1 in the ith row and jth column, and k be a primitive element of the field $\mathfrak{F}_q$. If $n \geqq 5$ it is not difficult to show that the cosets $C\mathfrak{N}$ and $D\mathfrak{N}$ generate $\mathfrak{G}(n, q)$ if we take

$$(1) \qquad C = I + ke_{n-1,2} + e_{n1}, \qquad D = (-1)^n \left(e_{12} - e_{23} + \sum_{i=3}^{n} e_{i,i+1} \right).$$

The matrix C has period p and we have obtained the required pair of generators in the case $p = 2$. Hence let $p = 2t + 1$ and take

$$(2) \qquad A = C - 2(e_{11} + e_{22}), \qquad B = D + (-1)^n t(e_{n2} - ke_{n-1,3}),$$

so that A has period two. Then $B^{-1}AB = A_1 = I - 2(e_{22} + e_{33})$ and $(A_1A)^2 = I + 2ke_{n-1,2}$. Using this result it can be shown that the subgroup $\mathfrak{H}$ generated by the elements of $A\mathfrak{N}$ and $B\mathfrak{N}$ contains $I - (2e_{n-1,n-1} + e_{nn}) = J_n$, contains J_nA, and so contains $(J_nA)^2 = I + 2(ke_{n-1,2} + e_{n1})$. Then $\mathfrak{H}$ contains C, it is easy to show that $\mathfrak{H}$ contains D, and so $A\mathfrak{N}$ and $B\mathfrak{N}$ are the required generators.

The argument in the case above fails when $n = 2, 3, 4$ as there is too little space in the matrices to carry out the computations needed. However, when $n = 4$ and $q \neq 9$ we have shown that the selection

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[1] The research which yielded these results was supported in part by NSF grant G-4792.

$$C = I + ke_{41}, \qquad D = e_{12} - e_{23} + e_{34} + e_{41} \tag{3}$$

yield generators, as before, and then provide the pair required when $p=2$. If $p=2t+1$ we use

$$A = e_{11} - e_{22} + e_{33} - e_{44} + ke_{41}, \quad B = e_{12} - e_{23} + e_{34} - kte_{42}. \tag{4}$$

When $n=4$ and $q=9$ the group is generated by $A\mathfrak{N}$ and $B\mathfrak{N}$ where

$$\begin{aligned} A &= -e_{11} + e_{22} - e_{33} + e_{44} + k(e_{32}) + e_{41}, \\ B &= e_{12} - e_{23} + e_{34} + e_{41} + e_{42} + k(e_{33}). \end{aligned} \tag{5}$$

In the case where $n=3$ there is even less room to operate but we can take

$$C = I + k(e_{31}), \qquad D = e_{12} + e_{23} + e_{31}, \tag{6}$$

where C has period p. As before, if $p=2t+1$, we use

$$A = 2(e_{33}) - C, \qquad B = D - (kt)e_{32}, \tag{7}$$

where A has period two.

There remains the more difficult case where $n=2$. In this case the matrices

$$B = \begin{pmatrix} k & 0 \\ 0 & k^{-1} \end{pmatrix}, \qquad C = \begin{pmatrix} 0 & 1 \\ -1 & k \end{pmatrix} \tag{8}$$

generate if $q>3$. Then A and B will generate if we select

$$A = \begin{pmatrix} a & b \\ c & -a \end{pmatrix}, \tag{9}$$

and choose a, b and c so that A is unimodular and so that

$$AB^jA = \begin{pmatrix} 0 & d \\ -d^{-1} & f \end{pmatrix} \tag{10}$$

for some j. This can always be achieved. The case $n=2$, $q=2$ or 3 is contained, of course, in the tables of Brahana.

THE UNIVERSITY OF CHICAGO

THE ANNUAL MEETING IN CINCINNATI

The Sixty-fourth Annual Meeting of the American Mathematical Society was held in Cincinnati, Ohio on Tuesday, Wednesday and Thursday, January 28–30, 1958 in conjunction with the Annual Meeting of the Mathematical Association of America. All sessions were held at the meeting headquarters, the Sheraton-Gibson Hotel, with the exception of those on Wednesday afternoon which convened at the University of Cincinnati. The attendance was about 750, including 547 members of the Society.

The thirty-first Josiah Willard Gibbs Lecture was delivered by Professor H. J. Muller of the Zoology Department, Indiana University, at 8:00 P.M. on Tuesday in the Roof Garden of the Sheraton-Gibson. The title of the lecture was *Evolution by mutation.* The presiding officer was Professor M. H. Stone.

The Presidential Address was delivered by Professor R. L. Wilder on Wednesday at 2:00 P.M. in Wilson Auditorium with Professor Richard Brauer presiding. Professor Wilder's lecture was entitled *On a certain class of topological properties.*

By invitation of the Committee to Select Hour Speakers for Annual and Summer Meetings, there were two addresses. Professor Nelson Dunford spoke on *A survey of the theory of spectral operators* at 2:00 P.M. on Tuesday, with Professor I. E. Segal presiding. Dr. C. D. Papakyriakopoulos spoke on *Some problems on 3-dimensional manifolds* on Thursday, with Professor E. E. Moise presiding.

A tea for the attending mathematicians and their guests was given by the University of Cincinnati on Wednesday at 4:00 P.M. in the Faculty Dining Room of the Union Building on the campus.

There was a banquet in the Roof Garden of the Sheraton-Gibson starting at 7:00 P.M. on Thursday for members of the Society. This event was made possible by the generosity of the following companies: Avco Corporation, Cincinnati Gas and Electric Company, Cincinnati Milling Machine Company, General Electric Company, International Business Machines Corporation, Procter and Gamble Company, Union Central Life Insurance Company, and Western and Southern Life Insurance Company.

There was a total of sixteen sessions at which contributed papers were presented. Presiding officers were Professor Richard Brauer, Professor R. C. Buck, Professor R. H. Cameron, Professor Samuel Eilenberg, Professor A. E. Heins, Professor J. G. Hocking, Professor Leo Katz, Professor M. H. Martin, Dr. R. J. Nunke, Professor B. J.

Pettis, Dr. Ivar Stakgold, Professor Ernst Snapper, Professor E. H. Spanier, Professor A. E. Taylor, Professor R. M. Thrall and Professor George Whaples.

Abstracts of the papers will be published in April and June issues of the *Notices* of the American Mathematical Society.

The Council met on Tuesday afternoon, January 28, 1958.

The Secretary announced the election of the following ninety-nine persons to ordinary membership in the Society:

Dr. Z. B. Alterman, Weizmann Institute of Science;
Dr. R. C. Alverson, Boeing Airplane Company, Seattle, Washington;
Mr. L. C. Barajas, La Latino-Americana Seguros de Vida, Mexico;
Professor F. J. Beutler, University of Michigan;
Mr. A. T. Beyer, General Electric Company, Cincinnati, Ohio;
Professor Olivier Biberstein, Memorial University;
Mr. P. M. Blair, Humble Oil & Refining Company, Houston, Texas;
Professor Richard Blum, University of Saskatchewan;
Dr. M. K. Brachman, Northwest Oil Company, Dallas, Texas;
Mr. Donald Bratton, Control Instrument Company, Brooklyn, New York;
Dr. W. F. Brown CONVAIR, San Diego, California;
Mr. C. M. Bruen, International Business Machines Corp., Endicott, New York;
Mr. Fernand Charrier, Biard St. Varent, Deux Sevres, France;
Mr. D. A. Clarke, University of Western Ontario;
Mr. P. A. Clavier, Zenith Radio Corporation, Chicago, Illinois;
Mr. B. J. Cohen, National Cash Register Company, Dayton, Ohio;
Professor R. J. Cormier, Northern Illinois University;
Professor Russell Cowan, University of Florida;
Mr. L. F. Cremona, Combustion Engineering, Inc., New York, New York;
Mr. J. W. Davis, Jr., Radio Corporation of America, Los Angeles, California;
Mr. G. E. Decker, Lewiston Porter Central School, Youngstown, New York;
Mr. D. D. Dix, Harvard University;
Mr. E. L. Dubinsky, Burroughs Corporation, Philadelphia, Pennsylvania;
Mr. H. L. Farris, Black, Sivalls & Bryson, Inc., Tulsa, Oklahoma;
Dr. Melvin Ferentz, International Business Machines Corp., New York, New York;
Mr. J. F. Firkins, University of Miami;
Mr. H. J. Figge, Harry J. Figge and Associates, Inc., Des Moines, Iowa;
Professor R. C. Foster, Tri-State College;
Mr. R. B. Gray, Erie Resistor Corp., Erie, Pennsylvania;
Mr. Cyril Graze, Whitestone, New York;
Professor J. R. Hamilton, Long Island University;
Dr. W. W. Hammerschmidt, U. S. Air Force, Washington, D. C.;
Mr. W. J. Harris, University College, Ibadan, Nigeria;
Mr. J. G. Harvey, Tulane University;
Dr. G. M. Helmberg, University of Washington;
Dr. C. J. Himmelberg, III, Midwest Research Institute, Kansas City, Missouri;
Mr. R. E. Holladay, Sandia Corporation, Albuquerque, New Mexico;
Dr. Maurice Horowitz, Goodyear Aircraft Corporation, Akron, Ohio;
Professor J. M. Horvath, University of Maryland;
Mr. Taqdir Husain, Syracuse University;

Professor Cassius Ionescu Tulcea, Yale University;
Professor Herbert Izbicki, University of Idaho;
Professor J. A. Izzo, Jr., University of Vermont;
Dr. J. G. Jewell, The Martin Company, Baltimore, Maryland;
Professor A. J. John, New Bedford Institute of Technology;
Mrs. Mary J. Johnson, University of Miami;
Mr. H. J. Jones, El Camino College;
Mr. D. W. Kahn, Yale University;
Dr. Fred Krakowski, University of California, Davis;
Mr. H. O. Kreiss, The Royal Institute of Technology, Sweden;
Mr. R. G. Krutchkoff, Columbia University;
Mr. Gustav Kuerti, Case Institute of Technology;
Professor Nin Kumasawa, Tokyo University of Fisheries;
Dr. K. S. Kunz, Schlumberger Well Surveying Corp., Ridgefield, Connecticut;
Mr. R. G. Laatsch, University of Tulsa;
Mr. W. J. Leinbach, Wake Forest College;
Mr. J. T. Leverich, Harvard University;
Mr. H. H. Love, Jr., University of Delaware;
Mr. Franklin Lyman, Brooklyn, New York;
Mr. B. J. McDonald, U. S. N. Mine Defense Laboratory, Panama City, Florida;
Mr. W. T. McKinney, CONVAIR Astronautics, San Diego, California;
Mr. G. J. Maltese, Yale University;
Mr. J. R. Micklich, New Mexico Military Institute;
Mr. K. L. Miller, Brown University;
Dr. W. L. Miranker, Bell Telephone Laboratories, Murray Hill, New Jersey;
Mr. D. E. Muir, The Martin Company, Denver, Colorado;
Professor R. S. B. Ong, University of Michigan;
Professor Aris Phillips, Yale University;
Dr. G. H. Pimbley, Jr., Los Alamos Scientific Laboratory;
Mr. H. A. Pogorzelski, Mathematical Reviews, Providence, Rhode Island;
Professor R. M. Rankin, Missouri School of Mines;
Mrs. Joanna W. Schot, David Taylor Model Basin, Washington, D. C.;
Mr. R. F. J. Schulz-Arenstorff, Army Ballistic Missile Agency, Huntsville, Alabama;
Mr. J. F. Schwab, Arlington, Virginia;
Mr. P. T. Semm, Hallicrafters Company, Inc., Chicago, Illinois;
Professor R. D. Sheffield, University of Mississippi;
Mr. L. W. Small, Yonkers, New York;
Mr. H. A. Smith, Remington-Rand UNIVAC, Philadelphia, Pennsylvania;
Dr. J. H. Smith, American University;
Professor R. J. Smith, Northern State Teachers College;
Mr. W. S. Soar, Ballistic Research Laboratories, Aberdeen, Maryland;
Professor C. E. Sperry, Alderson-Broaddus College;
Mr. A. S. Stankovich, U. S. Air Force, Washington, D. C.;
Mr. B. K. Swartz, Massachusetts Institute of Technology;
Dr. Selmo Tauber, University of Kansas;
Professor F. L. Taylor, Southern Oregon College;
Dean F. M. Tiller, University of Houston;
Professor E. H. Tompkins, Jr., North Carolina State College;
Dr. Anthony Trampus, General Electric Company, Cincinnati, Ohio;
Dr. J. P. Tull, Ohio State University;

Mr. William Weisman, Physics Laboratory, New York, New York;
Professor J. R. Wesson, Vanderbilt University;
Mr. W. L. Wetmore, Corning Glass Works, Corning, New York;
Mr. B. E. Wicks, Aerojet-General Corporation, Sacramento, California;
Dr. J. R. Wilts, International Business Machines Corporation, Endicott, New York;
Dr. J. S. White, Minneapolis-Honeywell Regulator Company, Minneapolis, Minnesota;
Mr. D. C. Wold, Rice Lake, Wisconsin;
Dr. Basil Zacharkiw, Seton Hall University;
Mr. J. F. Zelle, Cleveland Broadcasting, Inc., Cleveland, Ohio.

It was reported that the following two hundred and seventy-three persons had been elected to membership on nomination of institutional members as indicated:

Brooklyn College: Mr. Murray Schechter, Mr. R. M. Warten.

Brown University: Mr. A. M. Duguid, Professor Ulf Grenander, Professor Michael Papadopoulos, Professor Eli Sternberg.

University of California, Berkeley: Mr. G. D. Chakerian, Mr. Richard Cleveland, Mr. M. D. Davis, Mr. E. M. Ellentuck, Mr. Vladimir Filippenko, Mr. T. E. Frayne, Mr. R. H. Homer, Mr. Hajimu Ogawa, Mr. Julius Smith, Mr. M. R. Swift, Mr. T. N. Tracewell, Mr. J. E. Weidlich, Mr. Mitsuru Yasuhara.

California Institute of Technology: Mr. P. L. Crawley, Mr. D. F. Rearick.

University of California, Los Angeles: Mr. D. G. Cantor, Mr. K. M. Ferrin, Mr. W. G. Strang, Mr. E. O. Thorp.

University of Chicago: Mr. Yasuo Akizuki, Mr. S. U. Chase, Mr. E. D. Davis, Mr. H. H. Gershenson, Mr. Martin Helling, Dr. Shin-ichi Izumi, Mr. Adam Koranyi, Mr. C. E. Linderholm, Mr. A. L. Liulevicius, Mr. Gunter Lumer, Mr. P. H. Monsky, Mr. K. R. Nagarajan, Mr. N. C. Petridis, Mr. G. N. Pinkham, Miss Eliana F. Rocha, Mr. Hwa Tsang, Mr. Stephen Wainger, Mr. N. D. Whaland, Jr.

University of Cincinnati: Mr. Hsi-ching Liu.

University of Colorado: Mr. C. E. Aull, Mr. A. A. Nafoosi, Mr. R. G. Thompson.

Columbia University: Mr. J. G. Glimm, Miss Mary E. Powderly, Mr. W. W. Saffern, Mr. Shigeaki Togo, Mr. Kalathoor Varadarajan.

Cornell University: Miss Nancy L. Hannye, Mrs. Louise Hay, Mr. Anatole Joffe, Mr. Birger Lovgren.

DePaul University: Mr. W. G. Witthoft.

Duke University: Miss Ellen E. Brauer, Mr. C. H. Horsfield, Mr. E. K. Johnson, Miss Martha E. Lawrence, Mr. R. W. Llewellyn, Mr. M. J. Saadaldin, Mrs. Margaret J. Stone, Miss Nancy M. Turnbull, Mr. L. H. Williams.

University of Florida: Mr. J. F. Andrus.

University of Georgia: Mr. C. W. Patty.

Harvard University: Mr. D. M. Bloom, Mr. Harvey Brooks, Mr. R. W. Cottle, Mr. E. G. Effros, Mr. Adam Kleppner, Mr. T. B. Knapp, Rev. P. J. Knopp, S.J., Mr. J. P. McCabe, Rev. V. M. Manjarrez, S.J., Mr. J. I. Richards, Mr. George Sadowdky, Mr. S. S. Shatz, Mr. P. R. Yale.

University of Illinois: Mr. K. W. Anderson, Miss Judith A. Blankfield, Mr. J. C. McCall, Mr. L. R. McMurray, Mr. E. M. Paul, Mrs. Nancy J. Poxon, Mr. J. H. Shelly, Mr. R. N. Townsend.

Illinois Institure of Technology: Mr. David Sachs.

Indiana University: Mr. D. L. Hanson, Mr. T. W. Ting, Mr. J. A. Wendland.

Institute for Advanced Study: Dr. J. F. Adams, Professor Also Andreotti, Mr. Pierre Cartier, Mr. Hans Grauert, Professor Michel Herve, Mr. Daniel Lacombe, Professor P. P. W. Lorenzen, Dr. Sibe Mardesic, Dr. A. I. Martin, Professor John Myhill, Dr. Dieter Puppe, Dr. W. F. Stoll, Mr. Lucien Waelbroeck.

Iowa State College of Agriculture and Mechanic Arts: Mr. I. D. Ruggles.

University of Kansas: Mr. T. J. Head, Mr. H. L. Nelson, Mr. D. E. Wilson.

Lehigh University: Mr. A. L. Hilt, Mr. C. W. Sloyer.

University of Maryland: Mr. James Conlan, Mr. G. R. Blakley, Mr. R. W. Carroll, Mr. R. C. Kline, Jr., Professor G. J. Rieger, Dr. E. P. Shelly, Mr. D. F. Templeton, Jr., Mr. G. T. Trotter.

Massachusetts Institute of Technology: Mr. J. L. Boal, Professor L. A. E. Carleson, Mr. R. J. Crittenden, Mr. H. I. Gross, Mr. M. D. McIlroy.

University of Miami: Miss Beverly L. Brechner, Miss M. J. Leslie Leitch, Mr. C. B. Stortz.

Michigan State University of Agriculture and Applied Science: Mr. B. H. Barnes, Mr. R. P. Grobe, Mr. F. J. Kosier, Mr. H. B. McClung.

University of Michigan: Mr. K. I. Appel, Mr. J. M. Beck, Mr. P. E. Bedient, Mr. T. G. Birdsall, Mr. W. D. Bouwsma, Mr. J. W. Brown, Mr. L. E. Claborn, Mr. R. L. Eisenman, Mr. L. R. Halsted, Mr. Jesse Hamman, Mr. R. R. Legault, Mr. R. E. Lewkowicz, Mr. W. D. Marsland, Jr., Mr. D. R. Schuette.

University of Minnesota: Mr. R. D. Adams, Mr. W. A. Dolid, Mr. E. O. Nelson, Mr. S. E. Spielberg, Mr. F. S. Van Vleck.

University of Nebraska: Mr. S. E. Bohn.

New Mexico College of Agriculture and Mechanical Arts: Mr. K. W. Lopp.

New York University: Miss Esther Rodlitz.

Northwestern University: Dr. M. P. Drazin, Mr. Samuel Pasiencier, Mr. S. C. Port.

Ohio State University: Mr. M. R. Hopkins, Mr. Joseph Kohler, Mr. L. R. McCulloh, Mr. M. V. K. Kenon, Mr. W. C. Nemitz, Mr. Robert Silverman.

Oklahoma State University: Mr. V. Seshadri.

University of Oregon: Miss Yvonne H. Cuttle, Mr. R. S. DeZur, Mr. R. P. Pakshirajan.

University of Pennsylvania: Mr. L. N. Bidwell, Mr. Louis Brickman, Mr. E. M. Brown, Mr. Hsin Chu, Mr. J. P. Clay, Mr. Neil Grabois, Miss Rora F. Iacobacci, Mr. Larry Mann, Miss Ruth M. Roberts, Mr. P. H. Sellers, Miss Catherine E. Shields, Mr. Paul Weinberg.

Pennsylvania State University: Miss Dorothy L. Bordner, Mrs. Mary L. Oliver.

Princeton University: Mr. B. J. Birch, Mr. R. T. Bumby, Mr. A. H. Clark, Dr. D. E. Cohen, Mr. Adrien Douady, Mr. S. E. Dreyfus, Mr. G. B. Hensel, Mr. Kuno Lorenz, Mr. L. C. Mejlbo, Mr. R. W. Ritchie, Mr. F. M. Sand, Mr. H. F. Thornton, Mr. T. H. Wonnacott.

Purdue University: Mr. B. J. Boyer, Mr. F. W. Carroll, Jr., Mr. S. A. Husain, Mr. J. E. Mack, Mr. Togo Nishiura, Mr. W. E. Thompson.

Queens College: Miss Marva B. Parks, Mrs. Relita N. Rivelle.

Rice Institute: Mr. A. A. Armendariz, Mr. Alan Wilson.

Rutgers University: Dr. E. R. Gentile, Mr. J. B. Herder, Mr. J. P. Jordan, Mr. J. C. Leavy, Jr.

College of Saint Thomas: Mr. D. W. Smith.

University of Southern California: Mr. J. W. Bergquist, Mr. Edwin Hirschhorn, Mr. D. R. Ryan.

Stanford University: Mr. C. K. Bradshaw, Mr. E. D. Callender, Mr. K. S. Davis, Mr. A. E. Hurd, Mr. W. R. Jones, Mr. L. H. Lanier, Jr., Mr. M. A. Seelye, Mr. D. R. Sherbert, Mrs. Betty J. Stone, Mr. Andrew Torok, Mr. L. M. Young, Mr. R. R. Weiss.

Syracuse University: Mr. S. D. Chatterji, Mr. G. F. Clements, Mr. T. S. Frank, Mr. N. J. Pullman, Mr. Eugene Rogot, Mr. J. P. van Alstyne,

University of Texas: Mr. W. D. L. Appling, Mr. E. L. Bethel, Mr. R. F. Jolly, Mr. W. A. Kirby, Mr. D. F. Riddle, Mr. J. T. White.

University of Toronto: Mr. Martin Eisen, Mr. R. K. Fosu, Mr. R. J. Pegis, Mr. Romas Mitalas, Mr. J. C. Taylor.

Tulane University: Mr. A. J. Hulin, Mr. John Selden.

University of Utah: Mr. D. A. Ford.

Vanderbilt University: Dr. Jose Gallego-Diaz.

University of Virginia: Miss Mary K. Huggin, Mr. J. G. May, Mr. W. G. May, Mr. P. E. McDougle, Miss Mary G. Rodreque.

University of Washington: Mr. J. G. Ceder, Miss Dorothy J. Christensen, Mr. A. W. Marshall.

Washington University: Mr. D. A. Higgs.

Wayne University: Mr. R. F. Pavley.

University of Wisconsin: Mr. G. D. Allaud, Mr. S. J. Doorman, Mr. Herbert Hetherington, Mrs. Ruth Iscol, Mr. D. R. McMillan, Jr., Mr. M. M. Nanda, Mr. G. M. Nielsen, Mr. R. A. Schutt, Mr. R. D. Sinkhorn, Miss Patricia A. Tucker, Miss Sandra Weinstein, Mr. C. R. B. Wright.

Yale University: Mr. C. A. McCarthy, Mr. Neill McShane, Mr. R. A. Scoville, Mr. P. C. Shields, Miss Dorothea K. Stillinger, Mr. W. W. Tait.

The Secretary announced that the following had been admitted to the Society in accordance with reciprocity agreements with various mathematical organizations: Deutsche Mathematiker-Vereinigung: Professor G. M. Koethe, Dr. Horst Lippmann, Professor Hans Rohrbach, Professor Robert Sauer; London Mathematical Society: Professor J. H. H. Chalk, Mr. G. E. H. Reuter, Professor W. W. Sawyer; Société Mathématique de France: Dr. Jacques Neveu, Mr. Francois Norguet, Professor Makoto Ohtsuka, Professor J. G. Teixeira; Svenska Matematikersamfundet: Mr. S. Y. Christofferson, Professor T. H. Ganelius, Professor L. V. Hormander; Wiskundig Genootschap te Amsterdam: Dr. Jacobus Verhoeff.

The following appointments by the President were reported: as a *Committee to Review the Arrangements between the Society and the Johns Hopkins Press*: Harish-Chandra, A. E. Meder, A. W. Tucker; as a *Committee on Expository Books*: Lipman Bers, S. Bochner, A. M. Gleason, E. J. McShane, Deane Montgomery; to the *Committee to Study the Economic Status of Teachers*: A. A. Albert, David Blackwell, J. W. Green, Richard Bellman (other members: Wallace Givens, C. H. Fischer, H. M. Schaerf); as a *Committee to Make Noninations for* *the Executive Committee for* 1958: S. S. Cairns, Chairman, Edwin Hewitt, B. O. Koopman, W. M. Whyburn; as an *Arrangements Committee for the Summer Meeting*, 1958: C. B. Thoman, Chairman, J. H. Curtiss, Philip Franklin, H. M. Gehman, F. B. Hildebrand, Norman Levinson, Hartley Rogers, Jr., R. D. Schafer; as a *Committee to Nominate Officers and Members of the Council for* 1959: George Mackey, Chairman, Herbert Busemann, Marshall Hall, B. J. Pettis, G. de B. Robinson; as members of *Committees to Select Hour Speakers* (terms to expire December 31, 1959): *Summer and Annual Meetings:* R. L. Wilder (Committee will consist of J. W. Green, Chairman, Einar Hille, R. L. Wilder); *Eastern Meetings*: D. C. Spencer, (Committee will consist of R. D. Schafer, Chairman, Warren Ambrose, D. C. Spencer); *Western Meetings*: Raoul Bott, (Committee will consist of J. W. T. Youngs, Chairman, R. C. Buck, Raoul Bott); *Far Western Meetings*: H. F. Bohnenblust (Committee will consist of V. L. Klee, Chairman, Charles Loewner, H. F. Bohnenblust); *Southeastern Meetings*: A. D. Wallace (Committee will consist of G. B. Huff, Chairman, O. G. Harrold, A. D. Wallace); to the *Applied Mathematics Committee* for a three year term beginning January 1, 1958: Brockway MacMillan, S. M. Ulam; as Chairman for 1958: Garrett Birkhoff (Committee to consist of Garrett Birkhoff, Chairman, R. E. Bellman, R. S. Burington, Shizuo Kakutani, Brockway MacMillan, S. M. Ulam); as *Chairman of the Committee on Translations from Russian*: J. V. Wehausen; reappointed to the Committee for a three year period beginning July 1, 1957: R. E. Bellman, Hans Samelson (Committee will consist of J. V. Wehausen, Chairman, R. P. Boas, R. E. Bellman, Irving Kaplansky, Hans Samelson); to the *Joint Committee on Places of Meeting for a three year term beginning January* 1, 1958: G. R. MacLane (Committee to consist of R. M. Thrall, Chairman, R. D. Schafer, G. R. MacLane); as *Tellers for the, election of officers and members of the Council for* 1958: W. G. Lister, Chairman, D. A. Buchsbaum, R. T. Shield.

The following appointments to represent the Society were reported: at the installation of Edwin Cameron Clarke as President of Geneva College: Professor Wray G. Brady; at the inauguration of Val H. Wilson as President of Skidmore College: Edwin Brown Allen; at the inauguration of Richard Glenn Gettell as President of Mt. Holyoke College: Neal McCoy; at the inaguration of S. W. Martin as President of Emory University: J. H. Wahab.

The Secretary reported that the following have accepted invitations to deliver hour addresses: at the University of Miami in November, 1957: Hans Rohrbach; at the Annual Meeting in Cincinnati, January, 1958: Nelson Dunford and C. D. Papakyriakopoulos; at Hunter College in February, 1958: Louis Nirenberg; at Stanford University in April, 1958: W. W. Rogosinski; at Columbia University in April, 1958: J. C. Moore and I. M. Singer; at Chicago in April, 1958: George Whaples.

The Council voted that the Summer Meeting in 1961 should be held at Oklahoma State University.

The Council voted to set Meetings at Evanston, Illinois in November, 1958; the University of Wisconsin, Milwaukee, in November, 1961; at Stanford, California, on April 18–19, 1958; at Durham, North Carolina, on November 28–29, 1958.

The Council voted to set Council meetings April 25, 1958, in New York, and at the Summer and Annual Meetings.

It was reported that Professor R. S. Pierce will serve as Acting Associate Secretary for the West Coast Area while V. L. Klee is abroad in 1958–1959.

The Report of the Editorial Committee of the Bulletin was received. The 1957 Bulletin contained only 425 pages; however it is anticipated that it will contain 625 pages in 1958, and this number was authorized.

With respect to the new department of short announcements in the Bulletin, two actions were taken. The editors were authorized to print the following announcement at the beginning of the new section: *The purpose of this department is to provide early announcement of significant new results, with some indications of proof. Although ordinarily a research announcement should be a brief summary of a paper to be published in full elsewhere, papers giving complete proofs of results of exceptional interest are also solicited.*

The Council also voted that the Editorial Committee of the Bulletin should have the right to reject papers communicated by Council members.

The Council voted to authorize the Proceedings to publish 1006 pages in 1958, and the Transactions to publish three volumes of approximately 550 pages each. It was also voted to recommend to the Trustees that four volumes of the Transactions be published in 1959 if this can be financed.

The Council voted to approve the recommendations of the Colloquium Editorial Committee that the Colloquium volumes by Jackson, Stone, and Paley-Wiener be reprinted, that Garrett Birkhoff be authorized to prepare a revision of *Lattice theory*, and that S. S. Chern be the Colloquium lecturer in 1960.

The Report of the Mathematical Reviews Editorial Committee was accepted. The Council recommended that the Editorial Committee look into what incentives could be offered to reviewers to increase the number of reviewers and quality of reviews.

The Committee on Russian Translations reported that 1500 pages were selected for translation in 1957, and that it anticipates about the same number in 1958.

The Invitations Committee for the 1958 Summer Institute on Surface Area reported that the Institute will be held at Bowdoin College, in Brunswick, Maine. The Council approved the recommendation by the Organizing Committee for Summer Institutes that an Institute in Number Theory be arranged in 1959.

The Applied Mathematics Committee recommended that a Summer Seminar in applied mathematics be held in 1959 entitled "Modern Physical Theories and Associated Mathematical Developments." This recommendation was approved by the Council.

It was voted that, subject to the concurrence of the Mathematical Association and the Society for Industrial and Applied Mathematics, the Joint Committee on Employment Opportunities be discharged and the Employment Register be handled by the Headquarters Office of the Society under the policy supervision of a committee consisting of one member from each organization. After the 1958 April Meetings, the Register will be displayed at Summer and Annual Meetings only.

The Council voted that the Society should become a member of the Conference Organization of the Mathematical Sciences, the other members of which are the Mathematical Association of America, the Association for Symbolic Logic, the Institute of Mathematical Statistics, the National Council of Teachers of Mathematics, and the Society for Industrial and Applied Mathematics. This organization replaces the Policy Committee for Mathematics, whose membership was the same. The purpose of the organization is to assist the member organizations in areas of common interest of several of them.

The Annual Business Meeting of the Society was held at 3:15 P.M. on Wednesday, January 29 in Wilson Auditorium at the University of Cincinnati. The Secretary reported briefly on the affairs of the Society.

At the annual election of officers, which was completed by mail ballot on November 10, 1957, approximately 2000 ballots were received, and the following were elected to office:

President Elect (one year), E. J. McShane.

Vice Presidents (two years), Garrett Birkhoff and N. E. Steenrod.

Associate Secretaries (three years), V. L. Klee, G. B. Huff, and J. W. T. Youngs.

Member of the Editorial Committee of the Bulletin (three years), E. E. Moise.

Member of the Editorial Committee of the Proceedings (three years), P. R. Halmos.

Member of the Editorial Committee of the Transactions and Memoirs (three years), Samuel Eilenberg, G. P. Hochschild.

Member of the Editorial Committee of the Colloquium Publications (three years), Salomon Bochner.

Member of the Editorial Committee of the Mathematical Reviews (three years), J. L. Doob.

Member of the Editorial Committee on Mathematical Surveys (three years), Irving Kaplansky.

Member of the Committee on Printing and Publishing (three years), E. F. Beckenbach.

Board of Trustees (five years), H. F. Bohnenblust.

Members-at-Large of the Council: R. H. Bing, L. H. Loomis, I. E. Segal, P. C. Rosenbloom, Walter Rudin.

The ordinary membership of the Society at the end of 1957 was 5711, including 580 nominees of institutional members and 31 life members. There are 140 institutional members and 3 corporate members. The total attendance of members at all 1957 meetings (including this Annual meeting) was 3304, including 2441 registrations of members of the Society. The number of papers presented was 657. There were 15 hour addresses, 1 Gibbs Lecture, 1 set of 4 Colloquium Lectures, 1 Presidential Retiring Address, 10 papers at an Applied Mathematics Symposium, and 11 papers at a Symposium on Game Theory.

In response to a question from the floor, the Secretary reported on the status of the Society's efforts to assist Dr. Paul Erdös to obtain a re-entry permit into the United States. It was reported that Dr. Erdös is in Canada, and that the Secretary has made all efforts to see that while there he has the service of the lawyer whom the Society has retained. It is not proposed to retain the lawyer after expiration of present arrangements.

J. W. T. Youngs,
Associate Secretary

THE FEBRUARY MEETING IN NEW YORK

The five hundred forty-third meeting of the American Mathematical Society was held at Hunter College in New York City on Saturday, February 22, 1958. The meeting was attended by about 150 persons including 140 members of the Society.

By invitation of the Committee to Select Hour Speakers for Eastern Sectional Meetings Professor Louis Nirenberg of New York University delivered an address entitled *On partial differential equations of elliptic type* at a general session presided over by Professor R. E. Langer. Sessions for contributed papers were held in the morning and afternoon, presided over by Professor Mary Dolciani and Dr. E. J. Taft.

Abstracts of the papers presented in person appeared in the February issue of the *Notices* of the Society. The remainder of the abstracts will appear in the June issue.

R. D. SCHAFER,
Associate Secretary

BOOK REVIEWS

Intégration (Chapter V). By N. Bourbaki. Actualités Scientifiques et Industrielles, no 1244, Paris, Hermann, 1956. 2+131 pp. 1600 fr.

This is the second of an as yet unannounced number of fascicles of Bourbaki's work on integration. The preceding fascicle, containing Chapters I–IV, was reviewed in this *Bulletin* vol. 59 (1953) pp. 249–255.

The title of Chapter V is *Intégration des mesures*. This refers to the following maneuver. Let X and T be locally compact spaces; let μ be a positive measure on T; for each $t\in T$, let λ_t be a positive measure on X. Subject to appropriate conditions on the mapping $t\to\lambda_t$, it follows that $\nu=\int\lambda_t d\mu(t)$ is a positive measure on X. Various specializations and applications of this idea lead to a number of important topics. Among the major items considered are the following.

(1) Let π be a mapping of T onto X; let g be a non-negative function on T; and let $\lambda_t=g(t)\epsilon_{\pi(t)}$ where ϵ_x is the measure generated by a unit mass point at x. This leads to topics in the theory of discrete measures.

(2) Let $X=T$, and let π be the identity map; then formally, $\nu=\int gd\mu$, and the Radon-Nikodym theorem emerges.

(3) Let $g(t)\equiv 1$; then ν is the image under π of μ, and the discussion turns to changes of variable in an integral.

(4) Integration with respect to ν generates an iterated integral, and this leads to the Fubini theorem.

This brief outline indicates roughly the content of the chapter and also the extremely elegant way in which the entire discussion is centered around the idea of integration of measures. Regarding the work as an isolated essay on selected topics, one cannot praise it too highly. The treatment is beautifully unified and smooth to the point of seeming effortless. It is a delight to read.

Unfortunately, when this reviewer tries to place the work in the literature of measure theory, he is led to some unfavorable reactions. For one thing, the authors relegate to very minor roles a number of items that many people consider important. Examples:

(a) The phrase "absolute continuity" appears (as far as the reviewer can find) only in a parenthetic remark in what the authors call the Lebesgue-Nikodym theorem (known to most as the Radon-Nikodym theorem). The idea of absolute continuity is not exploited to the full, and with the one exception noted it is not identified by name.

(b) Necessary and sufficient conditions for mean convergence are mentioned only in an exercise (§5, Exercise 22) and even there the treatment leaves something to be desired. It is not pointed out that the conditions are necessary, and it is not pointed out that the three conditions can be reduced to two.

More important than these and other detailed objections is the fact that the authors still conceal the connection between measure theory and set theory. As in Chapters I–IV, measures are still pictured almost exclusively as linear functionals on spaces of continuous functions. Regardless of the merits of this approach to the subject, to raise a student on a diet of that and nothing else will probably retard his ability to communicate with the outside world. And, extol the virtues of Bourbaki as you will, there still is an outside world.

The reviewer feels very strongly that textbook authors, be they Bourbaki or mere mortals, should realize that their task is to supplement the literature, not to supplant it.

M. E. MUNROE

Generators and relations for discrete groups. By H. S. M. Coxeter and W. O. J. Moser. Ergebnisse der Mathematik und Ihrer Grenzgebrete, New Series, no. 14. Berlin-Gottingen-Heidelberg, Springer, 1957. 8+155 pp. DM32.

It is refreshing to find a book that not only studies groups but also deals with many particular and interesting groups. The major theorems of group theory have substance only insofar as they apply to actual groups. The Mathematician with any feeling for groups will welcome this monograph and its rich display of groups of many kinds.

The monograph deals for the most part with finitely presented groups, that is groups G generated by a finite number of elements $R_1, R_2, \cdots, R_m$ subject to a finite number of defining relations $g_k(R_1, \cdots, R_m)=1$, $k=1, \cdots, s$. There are two main faces to the study of finitely presented groups. The obverse is the problem of studying the properties of a group defined by given relations. Among other things we wish to know if the group is finite and if so, what its order is. The reverse is the problem of finding a simple set of defining relations for a given group. Both these problems are studied in this monograph, and a variety of methods, mostly geometrical, are employed. Since the word problem for groups is unsolvable, we are relieved of the necessity of searching for an all embracing method and may enjoy the elegance of several diverse approaches.

A practical method of enumerating cosets is given. This method has the advantage that on completion it yields a permutation representation of the group. A typical application of this method consists in showing that if G is defined by

$$V_1^3 = V_2^3 = V_3^3 = (V_2V_3)^2 = (V_3V_1)^2 = (V_1V_2)^2 = 1$$

then the subgroup generated by V_1 and V_2 is of index 5.

A large part of the monograph is geometric in its orientation. Chapters 3, 4, and 5 deal with graphs, Cayley diagrams, crystallography, and fundamental groups of certain manifolds. The Cayley diagram of a group G is a graph with one vertex for each element g of G and a directed arc associated with a generator x joining the vertex g to gx, there being an arc for every element g and generator x. The fundamental groups of two dimensional manifolds are studied. The seventeen two dimensional space groups are given in full.

Groups with special kinds of defining relations are discussed at length. Thus the group defined by $R^h = S^m T^n = RST = (TRS)^q = 1$ is designated as (h, m, n, q). Fortunately a series of tables in the appendix lists these notations and the selections in which they appear.

The regular tessellations of a space are associated with a group. The group is finite if the space is elliptic, but is infinite if the space is Euclidean or hyperbolic. Chapter 5 is devoted to this subject.

There is a brief discussion of Burnside's problem. In this connection I wish to correct a statement attributed to me. The Burnside group of exponent 4 with two generators is of order 2^{12} and has the defining relations

$$\begin{aligned} A^4 = B^4 = (AB)^4 = (A^{-1}B)^4 = (A^2B)^4 = (AB^2)^4 \\ = (A^2B^2)^4 = (A^{-1}B^{-1}AB)^4 = (A^{-1}BAB)^4 = 1. \end{aligned}$$

My statement was that the relation $(A^{-1}B^{-1}AB)^4 = 1$ is superfluous, not that $(A^{-1}BAB)^4 = 1$ is superfluous.

A great deal of space is devoted to finding simple defining relations for a number of known groups. First the symmetric and alternating groups are treated, there being an extensive literature on this subject. The connections with braid groups are noted and discussed. An entire chapter is devoted to defining relations for the unimodular and linear fractional groups over finite fields with the Mathieu group on 11 letters thrown in for good measure.

The last two chapters are purely geometric in their approach. Chapter 8 treats the regular maps on multiply-connected surfaces, this investigation being similar to the study of regular tessellations except that the conditions are now topological rather than metric. Chapter 9 *Groups generated by reflections* includes a vast number of groups, in particular the Dickson-Chevalley finite analogues of the exceptional simple Lie groups.

MARSHALL HALL, JR.

Games and decisions: *Introduction and critical survey*. By R. Duncan Luce and Howard Raiffa. John Wiley and Sons, Inc., 1957. 19+509 pp. $8.75.

This is a book written by mathematicians but "aimed primarily at those readers working in the behavioral sciences." It "attempts to communicate the central ideas and results of game theory and related decision making models unencumbered by their technical mathematical details: thus, for example almost no proofs are included." Despite these remarks from the authors' introduction, the book deserves the attention of professional mathematicians, behavioral scientists or anyone else interested in finding out about the subject matter of its title. Of these various groups I strongly suspect it is the mathematicians who will most readily understand its contents.

The book is unique in many respects. It is the first on the subject which attempts to cover the whole field, a feat it succeeds in doing with almost astonishing comprehensiveness. The authors have read, digested and present here in a lucid manner virtually every idea on games and decisions that has been put forth since these objects became the subject matter of a "theory." Even the game theory specialist will in all likelihood find branches treated here with which he is not familiar (this reviewer is grateful to the authors for the sections on statistical decision making).

The material of the book is divided up according to the following scheme: a general introductory chapter on games, a chapter on utility theory and then another more technical chapter on games, describing the extensive and normal forms. There follow three chapters on two-person games, the first on the, dare we call it, "classical" zero-sum theory, the next two on nonzero sum games, first the noncooperative theory, then the cooperative. Already we note a sharp contrast with other books which have appeared since von Neumann and Morgenstern, and which have devoted all or almost all their attention to the zero-sum theory. The present authors hasten on to more unsettled and controversial parts of the theory which constitute their main object of study. Nevertheless, for the sake of completeness, the book includes no less than seven appendices to the two-person zero-sum chapter, covering such related topics as linear programming, infinite and sequential games among others. The next five chapters are devoted to n-person games and a presentation of the many concepts and theories which have been proposed for them. After this comes a chapter dealing with applications of the theory. The final two chapters are entitled *Individual decision making under uncertainty*, and *Group decision making.*

The book has novelty as well as comprehensiveness. At a conservative guess, half of the material it contains has not previously appeared in book form. Game theory is a rather diffuse subject, and the authors have done an admirable job of bringing under one roof many different ideas, mostly from mathematics and economics periodicals, and giving to this diversified material at least the semblance of unity. One of their unifying devices is consistent use of two approaches to the theories, the constructive approach: "Here is a theory. Let us investigate its properties," and the axiomatic approach: "Here are some desirable properties. Let us find a theory that has them."

We have described the book so far in its role as a survey of the subject, but this is only half of the story. Its subtitle reads: "Introduction and *Critical* Survey" (our italics) and accordingly, the book is about equally divided between presentation of material and critical analysis of it. Say the authors: "Our critical discussion and our examples are strongly colored—at least as far as a mathematician is concerned—by a social science point of view." Thus, along with straight information the reader will also be given the benefit of the authors' thinking on each subject. We shall comment briefly on this second aspect.

Game theory occupies a curious position among mathematical theories in that it is not intended to explain anything. In the language of the authors, ". . . we feel that it is crucial that the social scientist recognize that game theory is not *descriptive* but rather (conditionally) *normative.* It states neither how people do behave nor how they should behave in an absolute sense [I should hope not!] but how they should behave if they wish to achieve certain ends." Thus, by the authors' own statement, experimental verification is not to be expected. What then is to be one's basis in accepting or rejecting a proposed theory? The answer can only be that in the last analysis this is a subjective matter in which each person must decide for himself whether his Platonic ideal of a completely rational decision maker would or would not behave in accordance with a particular theory. The authors have given these matters considerable thought and their opinions are worth listening to. Nevertheless, as one theory after another is introduced and then taken to pieces, the reader may experience a certain sense of futility. As an illustration, Chapter 10 is entitled "ψ-stability" and concerns an interesting notion introduced by one of the authors to handle n-person games. After describing the theory in the first two pages, the authors proceed to demolish it with apparent relish for eight pages. There follows a section treating a special example which it turns out is introduced as a further illustration of the weakness of the theory. The positive results consisting of theorems which have been proved are included in fine print.

Actually it is not true that all of game theory comes down to "philosophical questions." There are instances, though admittedly few, where it is generally agreed that game theory gives the "right" answers. Perhaps the best illustration is the manner in which it settles the theory of bluffing as treated by von Neumann and Morgenstern and many others. This is a very special situation, true, and old stuff by now. Still, wouldn't it be a kindness to the reader to present at least one case in which the right theory applied to the right situation successfully explains a certain kind of behavior? Isn't this, after all, precisely the sort of thing the theory formulators are shooting at? Here, it would seem, is cause for celebration, yet this is one of the few points on which the book is silent. At times one almost feels that the main purpose of the theories is not to solve problems, but to provide grist for the critical mill, and that the construction and destruction of game theories has itself become a sort of super game. Game theory is at best a debatable tool for the solution of behavioral problems. Because of their zest for criticism the authors may have made it look even weaker than it really is.

We have a few lesser complaints. The book is at times unnecessarily verbose. As an illustration, in the chapter on two-person zero-sum games a whole section is devoted to *Compatibility of the pure and mixed strategy theories*. Surely any scientist, no matter how behavioral, would be satisfied with a few sentences to the effect that since a pure strategy may be considered as a special kind of mixed strategy, the latter theory includes the former. The trouble with belaboring the obvious is not only that it exhausts the reader, but makes it hard for him to determine which are the truly subtle points. Secondly, while the book contains a fair number of examples, I believe the authors could have been still more generous in this regard. For example, in presenting a half dozen or so different theories of cooperative two-person games, it would seem natural to take some single specific game and compare its solutions as prescribed by each of the theories. This, for some reason, is not done. Finally, in a book of this sort one expects some general concluding remarks. I was disappointed in not finding any discussion of such annoying but natural questions as, "have we gotten anywhere?", and "where is all this leading?". After presenting so much criticism on special topics, the authors really owe the reader some sort of general tying together of the pieces.

It is customary in these reviews to give a list of typographical errors as evidence of the conscientiousness of the reviewer. Let it be reported therefore that on page 315, line 2, the authors inadvertently toss a coil instead of a coin.

For our own concluding remark we return to our initial observation. The authors have made available and understandable a great mass of interesting material in a new field, an achievement that will surely earn them the admiration and gratitude of all who read their book.

DAVID GALE

Variational methods for eigenvalue problems. By S. H. Gould. Mathematical Expositions, no. 10, University of Toronto Press, Toronto, 1957. 14+179 pp. $8.75.

Until the publication of this book there had been no monograph at all on the study of variational eigenvalue problems, and it has always been hard for the student to get a suitable introduction to the subject. Hitherto, he has had to rely on expositions having the character of an engineering textbook and these have always been mathematically unsatisfactory.

In conformity with the other books of the Toronto series, the author does not presuppose much mathematical knowledge on the part of the reader and gives lengthy and detailed explanations whenevery they seem desirable. The nature of his subject, however, compels him to make use of the methods and fundamental concepts of Lebesgue integration and of the theory of linear transformations in Hilbert space, and it is therefore necessary for him to develop parts of these theories in very little space. These brief developments are satisfactory for the purpose of the book, but are of necessity incomplete.

In the introductory chapters, the author considers eigenvalue problems in finite dimensional space using such problems as that of the vibrating string as examples. The equivalence between the eigenvalue problem and the variational problem is carefully established and the Monotony principle on which the Rayleigh-Ritz and Weinstein methods depend is explained in detail. It is now necessary to develop the theory of Lebesgue integration and to construct the space L^2. The classical eigenvalue problems for membranes, plates, and rods are considered in a further chapter and another describes most of the Weinstein method in its original form as applied to the problem of the vibrations of a clamped plate, a numerical example being given as an illustration. The author next develops what he needs of the theory of completely continuous operators on a Hilbert space and the spectral theorem for self-adjoint operators of that class. A further chapter is devoted to the Weinstein-Aronszajn method of approximating eigenvalues of completely continuous self-adjoint operators, the passage from the spectrum of one intermediary problem to the next being given in some detail. A short chapter on the problem of the vibrating plate illustrates the method. The book ends with a rather difficult chapter on the application of the method to general differential problems.

Although no special preparation is needed for the reading of the book, a mediocre student will find it quite difficult. He is required to absorb the fundamental concepts of the theory of Lebesgue integration, Hilbert spaces, completely continuous operators, Hilbert-Schmidt operators, reproducing kernels, functional and pseudo-functional completions, and systems of stable and unstable boundary conditions for differential problems. It is an ideal book to put in the hands of the graduate student whose interest in applied mathematics has led him to believe that mathematics itself (i.e. real variables, linear spaces etc.) is of no use to him. Besides providing an excellent introduction to the subject at the level where such an introduction is most needed, the book will also be useful as a readable survey of the study of eigenvalue problems for all who are unfamiliar with the subject.

Since this is a first printing, there are a number of rather obvious misprints which will not impede the reader; at two or three places in the text there are certain oversights in the arguments themselves which may delay a student somewhat, but in no case are these very serious.

WILLIAM F. DONOGHUE, JR.

Numerical analysis. By K. S. Kunz. New York, McGraw-Hill, 1957. 15+381 pp. $8.00.

This fine text book should be useful in a first course in numerical analysis given to students with backgrounds of calculus and elementary differential equations.

The first seven chapters cover the ABC's: Finite Differences, Interpolation, Differentiation and Integration, Roots of Equations. Included here is an interesting and useful chapter on summation of series, Euler's transformation, and various summation formulae. Then follow several long chapters devoted principally to three topics:

solution of ordinary differential equations, simultaneous linear equations, and elliptic partial differential equations via partial difference equations. These topics are covered adequately if not liberally as far as an introductory course would be concerned. Two final chapters on parabolic and hyperbolic equations and integral equations are perfunctory.

The text is liberally strewn with worked examples, and includes many problems, some theoretical, and some requiring numerical work with desk machines. Round-off, truncation, and instability, the three devils of numerical analysis, are introduced in such a way as to drive the student from the paradise of infinitely precise computation, without plunging him into the hell of infinitely precise error analysis. There is no direct treatment of electronic computers as such, but here and there are interpolated remarks to the effect that such and such technique is or is not well suited to automatic computers. There are a good many references to the recent literature on numerical analysis, and this way, the student should get the feeling that numerical analysis did not come to an end with Horner. On the other hand, these references are spotty; many references that one would like to see are not present, and this reviewer is left with the impression that the author was not clear how much to update his original notes of 1947–1949 in the face of the recent flux in the field.

PHILIP J. DAVIS

Primzahlverteilung. By Karl Prachar. Berlin, Göttingen, Heidelberg, Springer-Verlag, 1957. 10+415 pp. DM 55. Bound DM 58.

This treatise is a distinguished sequel to Landau's monumental *Handbuch.* It contains a skillfully presented, up-to-date and extensive account of that recondite branch of number theory—the analytic theory of the distribution of primes. Within its ten chapters are incorporated many remarkable new results never previously treated in any book. Above all, here may be found: Selberg's improvement of the Viggo Brun sieve technique; various interesting results of Erdös on the difference of consecutive primes; the theorem that almost every even integer is representable as the sum of two odd primes; Tatuzawa's proof of Rodosskii's theorem on the distribution of primes in "short" arithmetic progressions; the best known error term in the prime number formula; the theorems of Hoheisel, Ingham and Tatuzawa on the difference of consecutive primes; Rodosskii's proof of the celebrated theorem of Linnik on the smallest prime in an arithmetic progression.

Because of the enormity of his program the author reluctantly abandons the ideal of absolute completeness. For example, he does not consider the so-called "large sieve" of Linnik and Rényi's refinement of it. He is thus compelled to omit Rényi's theorem that every large even integer can be written as the sum of a prime and an "almost" prime. Again, he does not develop the intricate apparatus required to adopt the Brun sieve to obtain estimates from below.

Despite its omissions the book marshals an imposing array of ideas and theories. Prominently featured are the researches of Vinogradoff and his school, which have transformed much of analytic number theory since the year 1934. The uncanny ingenuity, depth and power of the new methods continue to astonish us almost a quarter of a century after their inception!

It is now fifty years since the analytic theory of numbers was first presented as a systematic science by Edmund Landau in the two volumes of his *Handbuch der Lehre von der Verteilung der Primzahlen.* Its coverage of every phase of the subject as it was known at that time was all embracing. The results of later researches were amalgamated by Landau in his 1927 *Vorlesungen über Zahlentheorie.* In a joint tribute to Landau [J. London Math. Soc. vol. 13 (1938) pp. 302–310] Hardy and Heilbronn commented that these pioneering books "transformed the subject, hitherto the hunting ground of a few adventurous heroes, into one of the most fruitful fields of research of the last thirty years." The next twenty years have seen a continued upsurge of interest in analytic number theory, and several important books on the subject have appeared. We may mention, for example, the Cambridge Tracts of Ingham and Estermann, the monograph of Trost, Titchmarsh's book on the Riemann zeta function, Vinogradoff's book on trigonometric sums, the book of Hua on additive prime number theory, and the book of Tschudakoff on Dirichlet's L functions. But all of these later works, outstanding as they are, are restricted in scope. The literature of our subject has indeed been enriched with the appearance of the book under review, and it is especially welcome.

Wisely the author does not caricature either variation of the characteristic and inimitable Landau style. He eschews the majestic sweep of the *Handbuch* and the austere exactitude of the *Vorlesungen.* Nevertheless his presentation is masterful and scholarly in its own right. He starts from the beginning, establishes his prospectus, and synthesizes his material. The result is a fine book which, in spite of the great difficulty and complexity of the subject matter, is self-contained, balanced, thorough, meticulous and pleasantly written.

The contents of the book, with its wealth of theorems, can only be summarily indicated in this review. The book opens with a succinct historical exordium. Chapter One is concerned with preliminaries. The Tschebyscheff inequalities for $\pi(x)$, the number of primes not exceeding a real number x, are derived; estimates are obtained for various sums and products involving primes; and applications to the totient function and divisor function are given.

The author gives an illuminating account of Selberg's sieve method in Chapter Two. By removing certain restrictions inherent in the older Viggo Brun technique Selberg has developed the sieve into an extremely versatile and flexible instrument. In spite of its natural limitations the capabilities of the method have not been fully explored. In this book only upper bounds are derived. Among the many applications given by the author is Brun's striking theorem: if p' runs over the twin primes, then the series $\sum 1/p'$ is at most convergent. But the age old conjecture that there are an infinitude of twin primes remains a major challenge to the ingenuity of mathematicians! Goldbach's problem may also be attacked by Selberg's sieve. For instance, Selberg has proved that every large even integer can be written as the sum of two positive integers, one of which contains at most two and the other at most three prime factors. It is regrettable that the author only alludes to Rényi's method for establishing lower bounds. Using his method Rényi in 1948 proved that there are infinitely many primes differing from "almost" primes by two. By an "almost" prime is meant a number containing no more than a fixed (but unspecified) number of prime factors.

The prime number theorem states that $\pi(x) \sim x/\log x$ and was first proved by Hadamard and de la Vallée Poussin in 1896. De la Vallée Poussin proved furthermore the refinement

$$\pi(x) = \int_2^x \frac{dt}{\log t} + R(x)$$

for sufficiently large x, with $|R(x)| < c_1 x e^{-c_2(\log x)^{1/2}}$, c_1 and c_2 being absolute positive constants. The proofs of both Hadamard and de la Vallée Poussin depend upon Hadamard's theory of entire functions, and in particular on the fact that $\zeta(s)$, apart from a simple pole at $s=1$, is regular all over the complex plane. In Chapter Three a simplified proof due to Landau is presented. The Hadamard theory, the Weierstrass product and the functional equation are not needed. Indeed the essential ingredient of the proof is the knowledge of a certain zero-free region of $\zeta(s)$ slightly to the left of the line $R(s)=1$. The author proceeds to the development of a body of theorems related to and logically interdependent with the prime number theorem.

Notable among these is the famous identity (Euler, von Mangoldt) of Landau's doctoral dissertation $\sum_1^\infty \mu(n)/n=0$. Recent discoveries have emphasized that the classification of a theorem of prime number theory according to depth is relative to the existing state of knowledge. Chapter Three concludes with the now almost classical elementary proof of the prime number theorem based upon Selberg's fundamental asymptotic formula. In this connection the author adroitly gives star billing to both Erdös and Selberg. To establish Selberg's formula the author follows the simplified proof of Iseki and Tatuzawa; he apparently has overlooked the fact that essentially the same proof was first published by Shapiro [Ann. of Math. (2) vol. 50 (1949) pp. 305–313]. The passage to the prime number theorem is then achieved via the method of R. Breusch.

It was to prove his ground-breaking theorem on primes in an arithmetic progression that Dirichlet in 1837 created his brilliant theory of characters. Let $k\geqq 1$ and $0\leqq l<k$, $(l, k)=1$. Then there exist infinitely many primes p such that $p\equiv l \pmod{k}$. The author, in Chapter Four, gives Dirichlet's proof as simplified by Mertens and Landau. He also sketches the elementary proof of Shapiro (1950). Let now $\pi(x, k, l)$ denote the number of primes p satisfying $p\leqq x$ and $p\equiv l \pmod{k}$. Employing the theory of the Dirichlet $L(s, \chi)$ functions de la Vallée Poussin proved the prime number theorem for arithmetic progressions. Speaking crudely we may say that he proved that in each of the $\phi(k)$ relatively prime residue classes modulo k there are "equally many" primes. More precisely he proved that

$$\pi(x, k, l) = \frac{1}{\phi(k)}\int_2^x \frac{dt}{\log t} + R(x, k, l),$$

with $|R(x, k, l)| < c_1 x e^{-c_2(\log x)^{1/2}}$, where $c_1=c_1(k)$, $c_2=c_2(k)$ are positive constants. For the proof of this theorem it is necessary to determine the behavior of $L(s, \chi)$ in the strip $0\leqq R(s)\leqq 1$; the zeros of $L(s, \chi)$ play a role analogous to those of $\zeta(s)$ in the study of $\pi(x)$. Note that the estimate of the error term $R(x, k, l)$ depends on k as well as x. In a series of researches initiated by Landau, Titchmarsh and Page the effect of the delicate interplay between k and x on the estimation of $R(x, k, l)$ has been investigated. There is, for example, an important theorem of Page which asserts that the above estimate of $R(x, k, l)$ is uniform in k for $k\leqq e^{c(\log x)^{1/2}}$ and x large unless k belongs to a certain set of exceptional values. Page's theorem has turned out to be an indispensable tool for handling many difficult problems of additive number theory [e.g. G. L. Watson's proof that every sufficiently large integer is the sum of seven positive cubes].

Its demonstration depends on the theory of L series as developed by Dirichlet, Riemann, Hadamard, de la Vallée Poussin, Hardy and Littlewood, Landau and others before the year 1935. The theorem of Page has subsequently been refined by Walfisz (1936). The proof of the theorem of Walfisz is based on Siegel's lower estimate of the value at $s=1$ of the $L(s, \chi)$ functions with real characters χ. The author gives Estermann's surprisingly simple proof of Siegel's result.

Sundry applications of the results in Chapters Two and Four are presented in Chapter Five. First, the sieve method is applied to the problem of obtaining bounds for $\pi(x, k, l)$. A typical result is the theorem of Titchmarsh. Let $0<a<1$ and $1 \leqq k \leqq x^a$. Then $\pi(x, k, l) < cx/\phi(k) \log x$, $c=c(a)$ for all l, $(l, k)=1$, $0 \leqq l<k$. An exact analogue of this inequality for lower bounds is not known. Density theorems are next studied. The theorem of Schnirelman that the integers representable as the sum of two odd primes have positive asymptotic density is proved. So is the theorem of Romanoff on the representation of integers in the form $p+a^m$. The theorems of Titchmarsh and Erdös on the number of divisors of $p-1$ are also derived. Finally, there is given a comprehensive treatment of the theory of the difference of consecutive primes. Included are theorems associated with the names of Knödel, Prachar, Erdös, Rankin, Chang, De Bruijn, Sierpinski and Walfisz.

The conjecture of Goldbach, enunciated by him in 1742, that every even integer greater than four is the sum of two odd primes, awaits proof or disproof. For almost two centuries this most difficult problem of additive number theory was intractable. In 1922 Hardy and Littlewood introduced the powerful Farey dissection method into analysis and proved on the basis of an unproved conjecture about the Riemann zeta function that every sufficiently large odd number can be represented as the sum of three odd primes, and that "almost" every even number is the sum of two primes. In 1937 Vinogradoff proved, in a sensational paper, the first of the Hardy-Littlewood theorems without employing any hypothesis. Later van der Corput, Estermann and Tschudakoff gave complete and independent proofs of the second theorem. Chapter Six is devoted to an exposition of Vinogradoff's method. It should be pointed out that Vinogradoff's result is in the nature of an existence theorem; it does not reveal explicitly a point beyond which all odd numbers are sums of three primes. It is therefore comforting to the reviewer to know that Rosser and Schoenfeld have proved (but have not yet published) the theorem that every odd integer with at least 350,000 digits is the sum of three odd primes.

In the long seventh chapter the author delves deeper into the function theoretic properties of the L functions. The successive section headings are: the functional equation; the partial fraction decomposition of $L'(s, \chi)/L(s, \chi)$; further theory of the distribution of zeros of $L(s, \chi)$; "explicit formulae" theorems; consequences of the Riemann hypothesis; another "explicit formula"; on the smallest prime in an arithmetic progression; irregularities of prime number distribution. The "explicit formula" theorems exhibit a curious type of connection, by exact infinite series representations, between the sums $\psi(x, \chi) = \sum_{n=1}^{x} \chi(n)\Lambda(n)$ and the zeros $\rho = \rho(\chi)$ of $L(s, \chi)$ in the critical strip. The principal results in the penultimate section are the least prime theorems of Chowla and Turan. A major theorem of the final section is the now classical result of Littlewood that $\pi(x) > li\ x$ infinitely often, where $li\ x$ is the logarithm-integral. Littlewood did not make any estimate of the smallest value of x for which the inequality is true. This problem has been studied by Littlewood's student Skewes over a period of many years. The latest (1955) result of Skewes is that the number does not exceed exp exp exp exp (7.705). Much earlier Hardy had already remarked, "However much the number may be reduced by refinements on Skewes's argument, it does not seem at all likely that we shall ever know a single instance of the truth of Littlewood's theorem."

Until 1921 the error term in de la Vallée Poussin's prime number theorem was the best known result of its type. In that year Littlewood employed the theory of Weyl sums to prove that $|R(x)| < c_1 x e^{-c_2(\log x \log \log x)^{1/2}}$. In 1935, Tschudakoff used Vinogradoff's abstruse method for estimating trigonometric sums to derive $|R(x)| < c_1 x e^{-c_2(\log x)^a}$, for a value of a between 1/2 and 1, and x large. We are indebted to Vinogradoff and Hua for the best result to date (1951):

$$|R(x)| < c_1 x e^{-\lambda(x)}, \qquad \lambda(x) = c_2 \frac{\log^{4/7} x}{(\log \log x)^{3/7}} \cdot$$

The proof given in Chapter Eight is based upon ideas of Tatuzawa. The crux of the argument is that the best known result concerning the region left of $\sigma = 1$ (with large t) in which $\zeta(s) \neq 0$ leads to the prime number theorem with a corresponding best known error term. To obtain such a zero-free region the method of Vinogradoff for dealing with the exponential sums arising in the approximate formulae for $\zeta(s, w)$ (Hurwitz zeta function) is used. Actually, the deeper theory of the zeta function is not required.

Tschebyscheff gave the first proof of Bertrand's postulate: if p_n

denotes the nth prime, then $p_{n+1} < 2p_n$. Legendre conjectured, but no one has ever proved, that $p_{n+1} - p_n < p_n^{1/2}$ for all sufficiently large n. Hoheisel in 1930 established the existence of a number a, $1-1/33000 < a < 1$, such that $p_{n+1} - p_n < p_n^a$. The exponent a was successively diminished by Heilbronn in 1933, by Tschudakoff in 1936 and by Ingham in 1937. Ingham obtained $a = 5/8$ and also a somewhat smaller value. The proofs of these and related results make use of the theory of the density of the zeros of the zeta function. In Chapter Nine the machinery of this theory is developed. Applications are given also to the work of Linnik (1943, 1945), Rodosskii (1949), Tatuzawa (1950) and Haselgrove (1951) on the distribution of primes in "short" arithmetic progressions. Further applications concern the estimation of $\zeta(1/2+it, w)$.

The crowning achievement of the last chapter is the deep theorem of Linnik: Let $k \geqq 2$, $(l, k) = 1$, $l < k$, and let $p_1(k, l)$ be the smallest prime in the arithmetic progression $nk + l$, $n = 1, 2, \cdots$. Then there exists a constant C independent of k such that $p_1(k, l) < k^C$. The awe-inspiring proof involves forty pages and twenty-one lemmas.

The book closes with an Appendix. This contains a brief summary of pertinent theorems and formulae from the theory of functions.

The author is to be congratulated for having written an important and valuable book. The House of Springer is to be congratulated on a superb example of the art of mathematical printing.

Albert Leon Whiteman

Neue topologische Methoden in der algebraischen Geometrie. By F. Hirzebruch. Ergebnisse der Mathematik und ihrer Grenzgebiete, New Series, vol. 9. Springer, 1956. 165 pp. DM 30.80.

This book, devoted to the topological transcendental theory of algebraic varieties over the complex field, should rank with Lefschetz's *L'Analysis situs et la géométrie algébrique*, Paris, 1924, and Hodge's *Harmonic integrals*, Cambridge, 1941, as a milestone in the development of the theory. While topology plays the essential rôle in Lefschetz's book and Hodge's main tool is harmonic differential forms, this book is characterized by the diversity of deep and difficult results which the author drew for his use. These include, among others, Todd's genus, Thom's algebra, and Kodaira's work on complex manifolds. Sheaves (or stacks or faisceau in French and Garbe in German) and analytic bundles with their characteristic classes are the pillars on which the main result is built.

The main result, which is not proved until the very end of the book, is the Riemann-Roch Theorem for nonsingular complex algebraic varieties. The classical Riemann-Roch Theorem is concerned with the following problem: On a compact Riemann surface there is associated to each meromorphic function f a divisor $\Delta(f)=\sum m_iP_i$, which is a formal sum of points P_i with integer coefficients m_i equal to the orders of f at P_i; m_i is positive if P_i is a zero and negative if P_i is a pole. In general, a divisor $D=\sum n_jQ_j$ is a sum of points Q_j in which only a finite number of the cofficients n_j are nonzero. The sum $\deg (D)=\sum n_j$ is called the degree of the divisor. A divisor is said to be $\geqq 0$ if all $n_j \geqq 0$. Divisors can be added and subtracted in an obvious way. Given a divisor $D=\sum n_jQ_j$, all meromorphic functions f on the Riemann surface such that $\Delta(f)+D\geqq 0$ form a complex vector space. (The condition means of course that f has a zero of order $\geqq -n_j$ at a point Q_j with $n_j<0$ and a pole of order $\leqq +n_j$ at a point Q_j with $n_j>0$ and is regular at all other points.) The classical Riemann-Roch Theorem says that the dimension of this vector space is

$$1+\dim |D| = \deg (D) - g + i + 1, \tag{1}$$

where g is the genus of the Riemann surface and $i-1$ is equal to the dimension of the divisor $K-D$, K being the canonical divisor. If D is ample, e.g., if D consists of a sufficient number of points with nonzero coefficients, then $i=0$ and formula (1) gives precise information on $\dim |D|$.

The author took (correctly) the view that this is really a theorem identifying two radically different numbers. Let M be an algebraic variety of dimension n and W an analytic vector bundle over M, having as structural group the complex general linear group $GL(q, C)$ in q variables. W will be called a line bundle if $q=1$. Denote by $\Omega(W)$ the sheaf of germs of holomorphic cross-sections of W and by $H^i(M, \Omega(W))$, $0\leqq i\leqq n$, the ith cohomology group. The most important of these cohomology groups is $H^0(M, \Omega(W))$, which is by definition the vector space of all global holomorphic cross-sections of the bundle W and which therefore contains some of the most valuable information on W. It is, however, the alternating sum

$$\chi(M, W) = \sum_{0\leqq i\leqq n} (-1)^i \dim H^i(M, \Omega(W)), \tag{2}$$

which has notable properties. For a constant coefficient sheaf the sum (2) gives the classical Euler-Poincaré characteristic.

The second number is related to the analytic structure in an entirely different way. In fact, let c_i, $1\leqq i\leqq n$, be the Chern classes of the tangent bundle of M, and d_j, $1\leqq j\leqq q$, be the Chern classes of W. Introduce formally the quantities γ_i, $1\leqq i\leqq n$, and δ_j, $1\leqq j\leqq q$, by the relations

$$(3)\qquad \begin{aligned} 1 + \sum c_i &= \prod (1 + \gamma_i), \\ 1 + \sum d_j &= \prod (1 + \delta_j). \end{aligned}$$

Define

$$(4)\qquad T(M, W) = \kappa_{2n}\left[(e^{\delta_1} + \cdots + e^{\delta_q}) \prod_{i=1}^{n} \frac{-\gamma_i}{\exp(-\gamma_i) - 1} \right],$$

where the expression inside the brackets is symmetric in γ_i and δ_j and hence can be expressed as a power series in c_i and d_j with rational coefficients, while the symbol κ_{2n} means the value of this cohomology class over the fundamental homology class of M. The Riemann-Roch-Hirzebruch Theorem says that

$$(5)\qquad \chi(M, W) = T(M, W).$$

In particular, this implies that $T(M, W)$ is an integer, which is by no means clear from the definition. If the bundle W is analytically a Cartesian product, both $\chi(M, W)$ and $T(M, W)$ depend only on M and we will denote them by $\chi(M)$ and $T(M)$ respectively. $\chi(M)$ is essentially the arithmetic genus of M and $T(M)$ is the Todd genus. By some analytic manipulation one can see that (5) will follow from the particular case

$$(6)\qquad \chi(M) = T(M).$$

But the proof of (6) is not less difficult than that of (5).

We mention in passing that (5) contains (1) as a particular case. In fact, a divisor on a Riemann surface defines a line bundle whose Chern class d_1 has the property that $\kappa_2(d_1)$ is equal to the degree d of the divisor. Formula (5) gives

$$(7)\qquad \dim H^0(M, \Omega(D)) - \dim H^1(M, \Omega(D)) = d - g + 1,$$

which is equivalent to (1) by Serre's duality theorem.

Returning to the proof of (6) it is natural to observe that the advantages are obvious if γ_i are actually cohomology classes. This is the case when M is a split manifold, which means that the tangent bundle is analytically equivalent to a bundle with the triangular group as structural group, or, in other words, there is an analytic field of flags (i.e., a sequence of linear subspaces $L_1 \subset L_2 \subset \cdots \subset L_n$ in the tangent space) over M. An important example of a split manifold is the bundle B of all flags over M. If we take the polynomial

$$(8)\qquad C(t) = 1 + \sum c_i t^i$$

with the Chern classes of M as coefficients, then this bundle B has the interesting property, reminiscent to the extension of an algebraic number field, that the inverse image of (8) in B decomposes into a product of linear factors. The author proves that $\chi(B)=\chi(M)$, $T(B)=T(M)$, so that it suffices to prove (6) for a split manifold.

The author's idea is to generalize $\chi(M, W)$, $T(M, W)$ to involve a a parameter. In the simplest case when the bundle W is not involved, they are defined as follows: Let Ω^p be the sheaf of germs of holomorphic p-forms over M. Then we define

$$\chi_y(M) = \sum_{0\leq i,p\leq n} (-1)^i \dim H^i(M, \Omega^p)y^p. \tag{9}$$

On the other hand, let

$$Q(y; z) = \frac{z(y+1)}{\exp(z(y+1)) - 1} + z \tag{10}$$

and

$$T_y(M) = \kappa_{2n}\left[\prod_{1\leq i\leq n} Q(y;\gamma_i)\right]. \tag{11}$$

The introduction of the parameter y combines several known invariants. In fact, by definition, $\chi_0(M)=\chi(M)$. The Hodge theory of harmonic differential forms shows that $\chi_{-1}(M)$ is the ordinary Euler-Poincaré characteristic of M and that $\chi_1(M)$ is the index of M. The latter is defined to be zero if n is odd and to be equal to the number of positive eigenvalues minus the number of negative eigenvalues of the intersection matrix of (real) dimension n of M, if n is even. From the Gauss-Bonnet formula for compact Kähler manifolds one can prove $\chi_{-1}(M)=T_{-1}(M)$. But in order to prove $\chi_y(M)=T_y(M)$ for all y, it is necessary and sufficient to prove their equality for a value of $y\neq -1$. The author's index theorem says that $\chi_1(M)=T_1(M)$, which therefore forms the bridge between the χ_y-theory and the T_y-theory.

The index theorem, which can be described as equating the index of a compact oriented manifold to a certain Pontryagin number, is a consequence of Thom's algebra. Its origin can be traced to a result of Pontryagin, which says that the index of a bounding manifold (i.e., one which is the boundary of a manifold of one dimension higher) is zero. Thom realized that in studying the characteristic numbers of compact manifolds it is important to introduce an equivalence relation, by calling two manifolds equivalent, when their difference is a bounding manifold. Defining sum as union and product as the Cartesian product, such equivalence classes of manifolds are made into an algebra. Thom proved that the tensor product of this algebra with the field of rational numbers has as a multiplicative base the complex projective spaces of different dimensions. The index theorem follows from the Thom theory by the simple observation that it is true for the complex projective spaces.

The above is just a description of some of the important ideas which enter into the proof, with no attempt to give an outline. The use of the index theorem is effective, but probably unnatural. Even if a more direct proof of the Riemann-Roch Theorem is found, many of the ideas in this book should be found useful in other problems. The field certainly deserves further study. In fact, the two natural directions of extension are either algebraic varieties over finite fields and with singularities allowed or general complex manifolds.

Because of the presence of the high-dimensional cohomology groups in (5) it may be felt that the theorem is probably less effective in applications than the classical case. For algebraic surfaces ($n=2$) the theorem implies the classical Riemann-Roch inequality and is sufficient for most purposes. (An exception is the theorem that if the plurigenera P_4 and P_6 are zero, then the surface is birationally equivalent to a ruled surface. For the proof of this theorem, the mere knowledge of the inequality is not sufficient; it is necessary to have some information about the superabundance of certain linear systems $|D|$, i.e., precisely about the value of dim $H^1(M, \Omega(D))$.)

The book uses many of the deep results in different branches of mathematics, and may cause difficulty even to readers with a good background. One should realize, however, that this is essentially an original paper. For such the introductory material is ample; it is also well written. If the reader succeeds in reaching the summit, the panorama is highly recommendable.

S. S. Chern

RESEARCH PROBLEMS

9. Olga Taussky.

It follows, e.g. from Burnside's basis theorem (see e.g. P. Hall, Proc. London Math. Soc. vol. 36 (1932) p. 35), that in a finite p-group G with derived group $G' \neq 1$, the quotient group G/G' cannot be cyclic. It was shown by O. Taussky (J. London Math. Soc. vol. 12 (1937)) that for 2-groups with G/G' of type (2,2) the derived group G' is cyclic and hence that $G'' = 1$. It is a difficult problem to estimate the length of the chain of successive commutator subgroups if the type of G/G' is given. The next cases to be studied are the types (2,4), (2,2,2) and (3,3). (See also W. Magnus, Math. Ann. vol. 111 (1935) and N. Itô, Nagoya Math. J. vol. 1, 1950.) (Received December 27, 1957.)

10. Olga Taussky.

Not every matrix of determinant 1 with integral rational elements is a commutator of matrices of the same nature. (See L. K. Hua and I. Reiner, Trans. Amer. Math. Soc. vol. 71 (1951).) What are necessary and sufficient conditions for a matrix of determinant 1 with rational integral elements to be a commutator of (1) integral matrices with determinant ±1, (2) integral matrices with determinant 1. (Received December 27, 1957.)

11. Olga Taussky.

A number of similar theorems are known for matrices with positive elements (positive matrices) and for positive definite symmetric matrices, but for which the available proofs are different. Can a unified treatment be given for both cases? Four examples of such theorems are:

1. The dominant eigenvalue exceeds the diagonal elements.
2. The intervals spanned by the quotients $\sum_k a_{ik}/x_i$ for a positive vector $x_1, \cdots, x_n$ include the dominant eigenvalue of a positive matrix; the intervals spanned by the quotients $\sum_k a_{ik}/x_i$ for an arbitrary non-null vector include an eigenvalue of a symmetric matrix.
3. The inequality

$$\det_{i,k=1,\cdots,n}(a_{ik}) \leqq \det_{i,k=1,\cdots,p}(a_{ik}) \cdot \det_{i,k=p+1,\cdots,n}(a_{ik})$$

for matrices with non-negative minors of all orders and for positive definite symmetric matrices. (This was pointed out to K. Fan who already found a unified treatment).

4. A matrix with all its minors of all orders non-negative has all eigenvalues real and non-negative; a positive semi-definite symmetric matrix has all eigenvalues real and non-negative. (Received December 27, 1957.)

DOCTORATES CONFERRED IN 1957

The following are among those who received doctorates in the mathematical sciences and related subjects from universities in the United States and Canada during 1957. In each case, the university, the month in which the degree was conferred, minor subjects (other than mathematics), and title of the dissertation are given.

Note that there is included at the end of these Notes a list of doctorates conferred in 1956 but not included last year.

A. H. Aaboe, Brown University, June, *On Babylonian planetary theories.*

W. R. Abel, University of Missouri, June, *Metric properties of metric arcs.*

I. J. Abrams, University of California, Berkeley, *Contributions to the stochastic theory of inventory.*

S. P. Agarwal, University of California, Berkeley, January, *On the asymptotic equivalence of two classes of tests of a multiparameter hypothesis.*

J. H. Alexander, The University of Tennessee, June, *Approximately finite geometries and their coordinate rings.*

R. L. Alonso, Harvard University, June, *A special purpose digital calculator for the numerical solution of ordinary differential equations.*

R. C. Alverson, Brown University, June, *Elastic-plastic beam dynamics by the method of characteristics.*

J. J. Andrews, University of Georgia, June, *Generalized winding numbers and light open maps.*

P. M. Anselone, Oregon State College, June, minor in French; *Convergency of the Wick-Chandrasekhar approximation technique in radiative transfer.*

D. N. Arden, Purdue University, June, minor in physics, *Curvatures of Riemannian manifolds.*

W. O. Ash, Virginia Polytechnic Institute, *Randomized estimates in power spectral analysis.*

Silvio Aurora, Columbia University, May, *Multiplicative norms of metric rings.*

Joseph Auslander, University of Pennsylvania, June, *Mean-L-stable systems.*

G. E. Bartsch, Harvard, *Confidence intervals for the means of non-normal populations.*

H. S. Bear, Jr., University of California, Berkeley, June, *Complex function algebras.*

D. A. Beckwith, Brown University, June, *An approximation method for plane gas flows.*

W. C. Bennewitz, University of Illinois, June, minor in Philosophy, *On Minkowski area.*

W. S. Bicknell, University of Michigan, June, *Premiums and reserves in multiple decrement theory.*

C. M. Braden, University of Minnesota, December, minor in physics, *Harmonic, biharmonic and related partial difference equations.*

J. C. Bradford, The University of Oklahoma, August, *Topological vector lattices.*

Louis deBranges, III, Cornell University, June, *Local operators on Fourier transforms.*

D. W. Bressler, University of California, Berkeley, June, *Absolutely measurable sets.*

B. D. Bucher, Princeton University, April, *The recovery of intervariety information in incomplete block designs.*

R. C. Bzoch, Illinois Institute of Technology, June, *On the existence and properties of the Lane integral.*

Robert Canavan, New York University, June, *Necessary conditions for continuation and reflection principles for solutions of homogeneous linear partial differential equations with constant coefficients.*

R. L. Carter, The University of North Carolina, August, *New designs for the exploration of response surfaces.*

E. W. Cheney, Jr., The University of Kansas, June, *On Gauge functions.*

Y. T. Chou, Carnegie Institute of Technology, September, *The effect of turbulence on slider-bearing lubrication.*

D. L. Clark, Oregon State College, June, minor in physics, *The distribution of linear functionals of stochastic processes.*

Leonard Cohen, Columbia University, July, *On mixed single sample experiments.*

H. H. Corson, III, Duke University, June, minor in physics, *Systems of equations in a finite field.*

P. R. Culwell, The University of Texas, June, *Iterated Holmgren-Riesz transforms in two dimensions.*

R. T. Dames, University of Michigan, June, *Stability and convergence for a numerical solution of the Goursat problem.*

R. L. Davis, University of Michigan, June, *Lie and Engle modules and their relation to Burnside's problem.*

D. F. Dawson, The University of Texas, August, *Continued fractions with absolutely convergent even or odd parts.*

M. R. Demers, Brown University, June, *On Lebesgue area.*

T. G. Donnelly, The University of North Carolina, June, *A family of sequential tests.*

P. H. Doyle, III, The University of Tennessee, August, *Tame, finite complexes in three-space.*

C. E. Duncan, Stanford University, January, *The asymptotic behavior of certain trigonometric sums.*

J. D. Esary, University of California, Berkeley, *A stochastic theory of accident survival and fatality.*

R. E. Esch, Harvard University, June, *The instablity of an unbounded parallel flow.*

Solomon Feferman, University of California, Berkeley, June, *Formal consistency proofs and interpretability of theories.*

J. E. Forbes, Purdue University, January, minor in physics, *Noncommutative finite groups with no irreducible representations of dimension higher than two.*

W. R. Fuller, Purdue University, August, minor in physics, *Existence theorems for periodic solutions of systems of differential and differential-difference equations.*

A. E. Garrett, Virginia Polytechnic Institute, *Estimation problems connected with stochastic processes.*

A. M. Garsia, Stanford University, September, *On surfaces with a rectilinear geodesic circle.*

J. M. Gary, University of Michigan, February, *Dualities in generalized manifolds and higher dimensional cyclic element theory.*

Betty J. Gassner, New York University, February, *On braid groups.*

W. E. Gibson, Massachusetts Institute of Technology, September, minor in physics, *Theory of unsteady laminar boundary layers.*

Ramanathan Gnanadesikan, The University of North Carolina, June, *Contributions to multivariate analysis including univariate and multivariate variance components analyis and factor analysis.*

A. J. Goldman, Princeton University, January, *A Cech theory of fundamental groups and covering spaces.*

S. W. Golomb, Harvard University, June, *Problems in the distribution of the prime numbers.*

N. R. Goodman, Princeton University, June, *The joint estimation of the spectra, cospectrum and quadrature spectrum of a two-dimensional stationary Gaussian process.*

Roderick Gould, Harvard University, June, *The application of graph theory to the synthesis of contact networks.*

J. W. Gray, Stanford University, September, *A theory of pseudogroups with applications to contact structures.*

D. S. Greenstein, University of Pennsylvania, June, *Derivative manifolds and Taylor series in the mean.*

Geoffrey Gregory, Stanford University, *An economic approach to the choice of continuous sampling plans.*

R. R. Haering, McGill University, May, *The electric and magnetic properties of graphite.*

Susan Hahn, New York University, June, *Stability criteria for difference schemes.*

A. B. Hajian, Yale University, June, *Measurable transformations and invariant measures.*

D. W. Hall, Princeton University, January, *Vacuum expectation values.*

A. E. Hallerberg, University of Michigan, June, *The development of the geometry of the fixed-compass with especial attention to the contribution of Georg Mohr.*

Oma Hamara, Massachusetts Institute of Technology, June, minor in physics, *Quadratic forms on infinite dimensional vector spaces.*

Mary I. Hanania, University of California, Berkeley, June, *Some statistical tests of hypotheses in learning theory.*

D. K. Harrison, Princeton University, January, *On torsion free Abelian groups.*

H. A. Heckart, Iowa State College, June, *Bursting speed of rotating discs.*

R. L. Helmbold, Carnegie Institute of Technology, June, *Semi-discrete potential theory.*

L. H. Herbach, Columbia University, June, *Optimum properties of analysis of variance tests based on Model II and some generalizations of Model II.*

N. J. Hicks, Massachusetts Institute of Technology, February, minor in humanities, *On the curvature and torsion of affine connexions.*

C. J. Himmelberg, III, University of Notre Dame, June, *A new approach to absolute retracts and absolute neighborhood retracts.*

S. P. Hoffman, Jr., Yale University, June, *Second-order linear differential operators defined by irregular boundary conditions.*

A. L. Hopkins, Jr., Harvard University, June, *An investigation of non-ohmic resistive switching networks.*

W. B. Houston, Jr., Massachusetts Institute of Technology, September, minor in physics, *Curvature and torsion of fiber bundles.*

H. V. Huneke, The University of Oklahoma, June, minor in physics, *Some properties of the Nörlund methods as an ordered set.*

L. C. Hunter, University of Oregon, June, *On induced topologies in quasi-reflexive Banach spaces.*

Mohammad Iqbal, The University of North Carolina, June, *On the classification statistic of Wald.*

R. B. Jackson, Jr., Duke University, June, minor in education, *Dependent solutions.*

W. D. James, University of Illinois, June, minor in physics, *Some potential representations in the plane.*

J. G. Jewell, University of Pittsburgh, February, *Essentially-convergent sequences and series.*

E. S. Johnson, The Catholic University of America, June, minors in physics and education, *Properties of solutions of nonlinear differential equations.*

J. R. Johnson, Jr., Duke University, June, Minor in economics, *Congruence properties of certain difference equations.*

M. W. Johnson, Massachusetts Institute of Technology, September, minor in mechanical engineering, *On the theory of thin elastic shells.*

P. G. Johnson, University of Minnesota, *Shares of functions with values in a Banach algebra.*

G. H. Josie, Johns Hopkins University, *Sampling variations as a factor in morbidity survey design.*

R. P. Kanwal, Indiana University, June, minor in physics, *Shock and wave surfaces for three-dimensional rotational gas flows.*

M. L. Keedy, University of Nebraska, June, minors in physics and philosophy, *Some properties of basic order relations in arithmetic of relation algebras.*

R. P. Kelisky, The University of Texas, *Interrelations between the numbers of Bell, Bernoulli, Euler, Fibonacci and Lucas.*

May E. R. Kinsolving, Syracuse University, June, *On spaces suitable for Finsler geometries in the large.*

T. C. Kipps, University of California, Berkeley, June, *The parametric problems for double integrals in the calculus of variations.*

J. E. Kist, Purdue University, June, *Topological ordered linear spaces.*

J. K. Knowles, Massachusetts Institute of Technology, June, minor in physics, *Thin elastic helicoidal shells.*

C. W. Kohls, Purdue University, June, *Ideals in rings of continuous functions.*

Walter Koppelman, New York University, June, *The Riemann-Hilbert theorem for finite Riemann surfaces.*

C. Y. Kramer, Virginia Polytechnic Institute, June, *Factorial treatments in incomplete block designs.*

H. C. Kranzer, New York University, February, *Water waves produced by surface disturbances.*

A. W. Kratzke, Oregon State College, June, minor in physics, *First passage time distributions in electronic receivers.*

Herbert Kurss, New York University, February, *The solution to some turning point problems.*

A. E. Labarre, Jr., The University of Oklahoma, June, minor in physics, *Differential calculus in Hilbert spaces.*

W. M. Laird, University of Pittsburgh, June, *An analysis of continuous beam-columns with uniformly distributed axial loading.*

J. W. Lamperti, California Institute of Technology, June, minor in philosophy, *On the asymptotic behavior of recurrent and "almost recurrent" events.*

H. J. Landau, Harvard University, June, *On canonical conformal maps of multiply connected regions.*

W. E. Langlois, Brown University, June, *Steady flow of slightly visco-elastic fluids.*

H. C. Lefkovits, The Rice Institute, May, *Some applications of adherent series.*

C. E. Leith, Jr., University of California, Berkeley, June, *One-dimensional perturbation of unbounded self-adjoint operators.*

C. J. Lewis, Brown University, June, *The problem of Milloux for functions analytic in an open annulus.*

Eric Liban, New York University, June, *On mixed boundary value problems for elliptic equations in the plane.*

H. M. Lieberstein, University of Maryland, February, minor in aeronautical engineering, *On the generalized radiation problem of A. Weinstein.*

J. C. Lillo, Princeton University, June, *Almost periodic solutions of ordinary differential equations.*

H. G. Loomis, The Pennsylvania State University, June, *On a method of solving some partial differential equations.*

K. R. Lucas, The University of Kansas, June, *Submanifolds of dimension* $(n-1)$ *in* E^n *with normals satisfying a Lipschitz condition.*

Edith H. Luchins, University of Oregon, June, *On some properties of certain Banach algebras.*

R. J. Lundegard, Purdue University, January, minor in psychology, *Identification and estimation in two stochastic models.*

I. S. Luthar, University of Illinois, June, *Uniqueness of the invariant mean on an Abelian semigroup.*

Leo Lynch, Virginia Polytechnic Institute, *On the analysis of paired ranked observations.*

J. J. Macdonnell, The Catholic University of America, June, minors physics and philosophy, *Some convergence theorems for Dirichlet-type series whose coefficients are entire functions of bounded index.*

W. S. Mahavier, The University of Texas, January, minor in physics, *A theorem on spirals in the plane.*

J. C. McCully, University of Michigan, February, *The operational calculus of the Laguerre transform.*

R. D. McWilliams, The University of Tennessee, minor in chemistry, June, *Extensions of linear transformations.*

Samuel Melamed, McGill University, October, *Differential equations near a singular point.*

William Mendenhall, North Carolina State College, *Estimation of parameters of mixed exponentially distributed failure time distributions from censored life test data.*

N. G. Meyers, Indiana University, September, minor in physics, *Asymptotic behavior of solutions of linear elliptic differential equations.*

D. W. Miller, University of Wisconsin, June, *The structure of uniform semigroups.*

P. D. Minton, North Carolina State College, *Some distributions related to column totals in sociometric matrices.*

Willard Miranker, New York University, February, *The asymptotic theory of solutions of* $\Delta u + k^2 u = 0$.

G. W. Morris, University of California, Los Angeles, June, *On the first and second boundary value problems of thermo-elasticity for an ellipsoid of revolution.*

R. D. Morrison, North Carolina State College, *Some studies on the estimates of the exponents in models containing one and two exponentials.*

W. O. J. Moser, University of Toronto, May, *Abstract groups and geometrical configurations.*

V. L. Mote, North Carolina State College, *An investigation of the effect of misclassification on the* χ^2 *tests in the analysis of categorical data.*

J. W. Neuberger, The University of Texas, June, minor in physics, *Continuous products and nonlinear integral equations.*

August Newlander, New York University, June, *Complex analytic structure for almost complex manifolds satisfying the integrability condition.*

J. A. Nickel, Oregon State College, June, minor in physics, *Plane wedge flow of a homogeneous incompressible isotropic Reiner-Rivlin fluid.*

A. E. Nussbaum, Columbia University, May, The *Hausdorff-Bernstein Widder theorem of semi-groups in locally compact Abelian groups.*

R. R. O'Brien, Massachusetts Institute of Technology, June, minor in physics, *A singular perturbation of a boundary value problem for a third order differential equation.*

E. S. O'Keefe, University of California, Berkeley, June, *On the independence of primal algebras.*

Norman Oler, McGill University, May, *An inequality in the geometry of numbers.*

M. W. Oliphant, The Catholic University of America, June, minors in physics and philosophy, *Singular sets and Hausdorff measures.*

Komorath Padmavally, The University of North Carolina, June, *A nonlinear integral equation.*

M. V. Pai, Purdue University, June, minor in general engineering, *Comparisons of the methods of classification.*

E. T. Parker, The Ohio State University, March, *On quadruply transitive groups.*

H. L. Pearson, Illinois Institute of Technology, June, minor in mechanics, *An extension of the concept of derivative with application.*

R. N. Pederson, University of Minnesota, June, minor in physics, *On the unique continuation theorem for certain elliptic partial differential equations.*

R. N. Pendergrass, Virginia Polytechnic Institute, *The rank analysis of triple comparisons.*

Alfredo Pinero-Perez, University of Michigan, February, *Légendre integral transforms.*

W. J. Perry, The Pennsylvania State University, August, *Mean value theorems for solutions of certain partial differential equations.*

W. J. Pervin, University of Pittsburgh, February, *Connected mappings of Hausdorff spaces.*

D. P. Peterson, University of Oregon, June, *Abstract Riemann sums on a compact group.*

Gideon Peyser, New York University, February, *Mixed initial and boundary problem of linear hyperbolic partial differential equations of higher order in two dimensions.*

D. L. Phillips, Purdue University, June, minor in physics, *Multiparabolic systems of partial differential equations.*

George Pimbley, New York University, June, *Solution of the initial value problem for the linearized multi-velocity transport equations with a slab geometry.*

Vera S. Pless, Northwestern University, June, *Quotient rings of continuous transformation rings.*

I. P. Polonsky, New York University, June, *Pseudo-analytic function with characteristic coefficients in* L_p.

H. L. Pond, The George Washington University, June, *Some relations between input and output power spectra in certain nonlinear systems.*

W. O. Portmann, Case Institute of Technology, June, *Hausdorff-analytic functions of matrices and linear transformations.*

G. C. E. Prins, University of Michigan, June, *On the automorphism group of a tree.*

S. E. Puckette, Yale University, June, *Semigroups and unbounded difference operators.*

M. O. Rabin, Princeton University, January, *Recursive unsolvability of group theoretic problems.*

James Radlow, New York University, June, *Diffraction of a dipole field by a unidirectionally conducting screen.*

E. R. Rang, University of Minnesota, December, minor in mechanics, *Periodic solutions of singular perturbation problems.*

R. R. Read, University of California, Berkeley, *Contributions to the statistical theory of cloud chamber data.*

H. J. Rebassoo, University of Wisconsin, June, *Constant breadth curves on the sphere.*

Dirk van der Reyden, North Carolina State College, *The use of orthogonal polynomial contrasts in the confounding of factorial experiments.*

R. H. Riffenburgh, Virginia Polytechnic Institute, *Linear discriminant analysis.*

D. S. Rim, Indiana University, June, *Axioms for the cohomology theory of finite groups and cup product.*

E. R. Rodemich, Stanford University, September, *Flows with free boundaries in a tube.*

J. I. Rosenblatt, Columbia University, *Goodness-of-fit tests for approximate hypotheses.*

A. I. Rosenfeld, Columbia University, April, *Specializations in differential algebra.*

M. J. Sane, University of California, Berkeley, *Locally unbiased tests of composite hypotheses with s contraints.*

D. W. Sasser, Yale University, June, *On Jordan matrix algebras.*

Martin Schechter, New York University, June, *On estimating partial differential operators in the L_2 norm.*

J. F. Schell, Indiana University, September, minor in physics, *Intrinsic deformation theory of a family of subspaces in a Riemann space.*

J. E. Scroggs, The Rice Institute, May, *On invariant subspaces of a normal operator.*

J. F. Seewerker, University of California, Los Angeles, June, *The extendability of Riemann surface.*

R. J. Semple, Princeton University, April, *Cohomology with unit coefficients in local fields.*

N. E. Sexauer, University of Illinois, October, *Some results concerning transformation rings which satisfy various density conditions.*

Patrick Shanahan, Indiana University, September, *An axiomatic characterization of the reduced homology theory.*

Lena Sharney, New York University, June, *Some results in combinatorial set theory.*

P. W. Shaw, Stanford University, September, *Variational methods in conformal mappings with applications to hydrodynamics.*

E. P. Shelly, Carnegie Institute of Technology, June, *Difference equations of polyharmonic type.*

F. A. Sherk, University of Toronto, November, *The theory of regular maps.*

M. M. Siddiqui, The University of North Carolina, June, *Distributions of some serial correlation coefficients.*

F. B. Sleator, New York University, February, *A variational solution to the problem of scalar scattering by a prolate spheroid.*

Daniel Slotnick, New York University, June, *Asymptotic behavior of solutions of canonical systems near a closed unstable orbit.*

Stephen Smale, University of Michigan, February, *Regular curves on Riemannian manifolds.*

D. L. Smith, The University of Rochester, June, *Uniqueness theory for Laguerre series.*

J. W. Smith, Columbia University, November, *Stability of quasilinear differential equations.*

M. B. Smith, Jr., The University of North Carolina, June, *An investigation of normal spaces.*

R. B. Smith, Yale University, June, *Representation of certain algebras on Hilbert space.*

J. H. Stapleton, Purdue University, June, minor in psychology, *On the theory of asymptotic distributions* (mod.) *and its extension to abstract spaces.*

N. B. Stein, Cornell University, June, *The third obstruction in complex projective spaces.*

G. R. Strohl, Jr., University of Maryland, August, *Peano spaces which are either strongly cyclic or two-cyclic.*

R. G. Swan, Princeton University, June, *Spaces with finite groups of transformations.*

E. W. Swokowski, University of Wisconsin, June, *On the theory of certain Lie rings of skew elements.*

M. F. Tinsley, The Ohio State University, December, *Permanents of cyclic matrices.*

Anthony Trampus, Case Institute of Technology, June, *Differentiability and analyticity of functions in linear algebras.*

Peter Treuenfels, New York University, June, *Exponential decay of solutions of certain linear partial differential equations.*

E. F. Trombley, University of Cincinnati, June, *Maximum theorems for Hilbert transforms in one and two variables.*

J. P. Tull, University of Illinois, June, minor in physics, *Dirichlet multiplication in lattice point problems.*

L. H. Turner, Purdue University, August, *The direct method in the calculus of variations.*

J. B. Tysver, University of Michigan, June, *Inherent errors in matrices with statistical applications.*

S. I. Vrooman, University of Pittsburgh, February, *Involutions in the plane of two complex variables.*

J. W. Walker, The University of North Carolina, June, *Optimal decomposition of a sample space for estimations based on grouped data.*

A. I. Weinzweig, Harvard University, June, *On fibre spaces and fibre bundles.*

E. F. Whittlesey, Princeton University, January, *Classification of finite 2-complexes.*

J. B. Wilson, University of Florida, June, minor in engineering mechanics, *An orthotropic circular disk subjected to its own weight when supported at a point.*

Eric Wolman, Harvard University, June, *Optimum difference operators for digital control systems.*

Maria J. Wonenburger, Yale University, June, *On the group of similitudes and its projective group.*

L. T. Wos, University of Illinois, June, minor in philosophy, *On commutative prime power subgroups of the norm.*

R. L. Yates, University of Florida, June, minor in philosophy, *On Waring's problem with cubic functions.*

J. J. Yeh, University of Minnesota, December, *Nonlinear Volterra functional equations and linear parabolic differential systems.*

Peter Yff, University of Illinois, October, minor in geology, *On line complexes in a projective four-space.*

L. N. Zaccaro, Syracuse University, June, *A problem-sequence development of introductory topology.*

P. A. Zaphr, University of Pittsburgh, June, *An examination of Euler methods of summability for double series.*

E. M. Zaustinsky, University of Southern California, June, *Spaces with nonsymmetric distance.*

David Zeitlin, University of Minnesota, August, *Behavior of conformal maps under analytic deformation of the domain.*

Alexis Zinger, University of Montreal, *On the choice of the best amongst three normal populations with known variances.*

The following doctorates were conferred in 1956, but were not included in the list in the preceding volume of this Bulletin (vol. 63, pp. 210–221):

A. W. Adler, University of California, Los Angeles, August, *Characteristic classes of homogeneous spaces.*

Olive J. Dunn, University of California, Los Angeles, August, *Estimation problems for dependent regression.*

R. L. Dunn, University of California, Los Angeles, August, *Set-valued functions without fixed points.*

D. A. Pope, University of California, Los Angeles, August, *The approximation of function spaces and some applications to the numerical analysis of problems in the calculus of variations.*

D. W. Robinson, Case Institute of Technology, June, *Functions of commutable linear transformations.*

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As a result of the splendid cooperation accorded to the project by most of the mathematicians and mathematical scientists who have received questionnaires to fill in, the mathematical section of the Register is now remarkably complete. However there are still a few gaps to be filled in.

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